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“New vistas in chaos”

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October 26, 2023 -- tutorials in Rm. 5209

https://us02web.zoom.us/j/82283409520?pwd=elo4Nlh5Q29aMEV2cXkxZ1VvQVZSdz09

- **10AM-noon Rainer Engelken (Columbia)**

Quantifying chaos, dynamical entropy and attractor dimensionality of spiking neural networks

The ergodic theory of chaotic dynamical systems allows us to characterize the dynamics and complex phase space structure of large-scale neural network models. In this tutorial, first, a brief introduction to the mathematical theory of chaos and strange attractors will be presented on the blackboard. This overview is designed to have broad applicability, useful for fields as diverse as machine learning, computational neuroscience, and other quantitative disciplines interested in dynamical systems. This will be followed by a numerical example illustrating the Lyapunov spectrum of a balanced spiking neural network of quadratic integrate-and fire neurons. Finally, we provide checks of the numerical implementation and introduce a novel algorithm for numerically exact, efficient, and event-based implementation of large, sparse spiking networks. The new algorithm reduces the computational cost per network spike from O(N) to O(log(N)). This enables numerically exact simulations of large spiking networks ($N=10^9$ neurons) and facilitates the characterization of their phase-space structure.

- **Noon-12:45: Lunch – please RSVP**
- **12:45-2:45 Enrique Pujals (GC-CUNY)**

“Back and forth on generic dynamics: hitchhiking on a phenomena/mechanism correspondence using geometric methods”

Abstract: Through geometrical models, perturbation techniques and sometimes analytical approaches, it has been possible to establish a dictionary between "a taxonomy of generic dynamical phenomenons" and a list of "simple mechanisms or dynamical configurations" responsible for such phenomenons. I will try to explain that approach in a general context and we will focus on the case of symplectic and Hamiltonian dynamics and in some cases, to their geodesic flows counterpart.

October 27, 2023 -- symposium in Rm. 4102

https://us02web.zoom.us/j/87656582525?pwd=RlRKUG8rQU5lT0FBeDdCTUg3RWlWZz09

10:30-noon Amit Vikram (UMD), Quantum ergodicity beyond random matrices

Noon-1 Lunch please RSVP

1-2:30 Bassam Fayad (UMD) KAM stability and rigidity: old and new
2:30-2:45 Coffee
2:45-4:15 Rainer Engelken (Columbia) Lyapunov spectra of recurrent neural networks: implications for ML

Amit Vikram (UMD):

"Quantum ergodicity beyond random matrices"
The fundamental assumption of statistical mechanics is that the long-time average of any observable becomes independent of the specifics of the initial state. In classical mechanics, this stems from Boltzmann's ergodic hypothesis, by which a generic initial state in an ergodic system visits the neighborhood of all states in phase space with the same energy. However, wavelike effects in quantum mechanics have made it difficult to identify what it even means for a quantum system to be ergodic, except on a case-by-case basis for individual observables.
In parallel, the "quantum chaos conjecture" suggests that "quantum chaos" should be associated with energy levels that statistically resemble the eigenvalues of random matrices. While this has been a useful observable-independent guiding principle (and is sometimes adopted as a "definition" of quantum chaos), several puzzles remain, including exactly what kind of "quantum chaos" the conjecture refers to.
In this talk, we will seek to address these two issues from first principles for a general quantum dynamical system, providing an overview of [1]. We will show that a discretized version of ergodicity can be quantized, without reference to specific observables, in terms of visiting the neighborhood of every state in a complete orthonormal basis in the quantum Hilbert space. We will also show that it is this quantum form of ergodicity that is quantitatively determined by the system's energy level statistics, with random matrix statistics emerging as a special case of a more general set of constraints. A key implication is that this kind of "quantum chaos" associated with energy levels has little to do with the popular understanding of chaos in terms of exponential instabilities or unpredictability.
[1] A. Vikram and V. Galitski. "Dynamical quantum ergodicity from energy level statistics." Phys. Rev. Res. 5, 033126 (2023). arXiv: 2205.05704 [quant-ph].

Rainer Engelken (Columbia)

"Lyapunov Spectra of Recurrent Neural Networks: Implications for Machine Learning"
We examine the dynamics of recurrent neural networks by calculating their full Lyapunov spectrum. Our results show a size-invariant Lyapunov spectrum and attractor dimensions smaller than the phase space dimensions. Through random matrix theory, we provide analytical approximations for the Lyapunov spectrum near the onset of chaos for strong coupling and discrete-time dynamics. We also uncover a point-symmetry in the Lyapunov spectrum, reminiscent of symplectic structures in chaotic Hamiltonian systems. For trained recurrent networks, our analysis serves as a quantitative measure of error propagation and stability. Based on these findings, we propose to mitigate the vanishing/exploding gradient problem by regularizing Lyapunov exponents, thereby highlighting the potential of dynamical systems theory in machine learning.

Bassam Fayad (UMD)

KAM stability and rigidity : Old and new