Quantum Field Theory

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Quantum Harmonic Oscillator

In Quantum Mechanics, the harmonic oscillator can be described by:

$$\begin{aligned}\hat{\mathcal{L}} &= \frac{1}{2}m(\frac{d\hat{q}}{dt})^2 - \frac{1}{2}k\hat{q}^2 \\ \hat{\mathcal{H}} &= \frac{1}{2m}\hat{p} + \frac{m\omega^2}{2}\hat{q}^2\end{aligned} \tag{1}$$

Then one obtains a familiar set of creation and annihilation operators:

$$\left[\hat{\alpha}, \hat{\alpha^{\dagger}}\right] = 1 \tag{2}$$

These operators act on the system by raising/lowering the energy level. The position in terms of these operators is given by:

$$\hat{q}(t) = \sqrt{\frac{\hbar}{2m\omega}}(\hat{\alpha}e^{-i\omega t} + \hat{\alpha}^{\dagger}e^{i\omega t}) \tag{3}$$

Quantum Field Theory

Quantum Field Theory was developed in order to combine Quantum Mechanics with Special Relativity. The entities of interest are quantum fields. Within the context of this theory one treats a field as a collection of infinite harmonic oscillators at each point in space.

$$\phi(\vec{x}, t) \sim \int d^3\vec{k}(\hat{\alpha}(\vec{k}) + \hat{\alpha}^\dagger(\vec{k})) \tag{4}$$

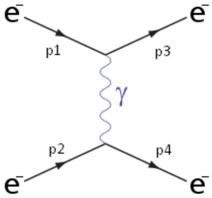
Contrast this with eq (3) for instance. The ladder operators in QFT create/annihilate entire particles. For instance the state:

$$|\vec{p}\rangle = \hat{\alpha}^\dagger(\vec{p})\,|0\rangle \tag{5}$$

describes one particle with 3-momentum $\vec{p}$ and energy $E = \sqrt{\vec{p}^2 + m^2}$. This is another advantage of QFT-it is a framework that can be used to describe processes in which the number of particles change! $(e^- e^+ \to 2\gamma)$

Types of particles in QFT

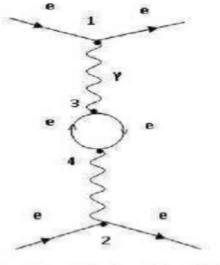
The main types of particles we are interested in are gauge bosons and fermions.
Fermions are the particles that matter is comprised of. In other words, leptons and quarks. Their main feature is that they possess half-integer spin and obey the Fermi-Dirac statistics.
Fermions interact with each other via four possible interactions-electromagnetic, weak, strong and gravitational. Withing the context of QFT, each of these interactions is said to be mediated by a gauge boson. For the electromagnetic interaction for example is the photon, γ.



Moller scattering

Interactions and Feynman diagrams

Having introduced the necessary components, one then would like to study the interactions of these fields, in other words the scattering processes. These can be very efficiently studied with the use of Feynman diagrams. The strength of an interaction is measured by a coupling constant. For the electromagnetic interaction for example, it is the fine-structure constant (or equivalently the electric charge $\alpha = \frac{e^2}{4\pi}$. In a Feynman graph these are typically carried by the vertices. For the Moller scattering we saw earlier, the graph is of order $\mathcal{O}(e^2)$. Another process contributing to this scattering is given by:

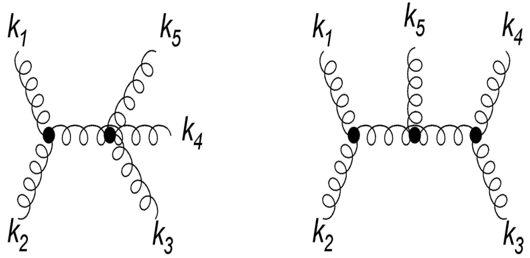


$$=\sim \mathcal{O}(e^4)$$

Moller scattering at 1-loop order

Interactions and Feynman diagrams

If the coupling constant of the theory is sufficiently small (≪1), then one can use an expansion in terms of it and ignore more complicated processes without sacrificing much in terms of accuracy. This is for instance the case of QED. On the other hand, if the theory is strongly coupled then that is no longer the case and one must include all the graphs, the calculation of which can be quite complicated.



Interactions and Feynman diagrams

Summarizing:

- In Quantum Field Theory, fields are treated as an infinite set of oscillators. The relevant operators, in analogy with the quantum harmonic oscillator, create/annihilate entire particle states. These correspond to the familiar particles, fermions and gauge bosons-matter particles and force propagators.
- Particles interact and the strength of these interactions is quantified by the coupling constant. The scattering processes resulting from these interactions are described by Feynman diagrams.
- For weakly coupled theories (QED), we can expand a process in terms of the coupling constant and ignore terms of higher order. For strongly coupled theories (QCD) all diagrams contribute significantly and must be taken into account.

The purpose of the next section will be to introduce some tools which help facilitate these calculations.