Quantum chaos versus many-body localization

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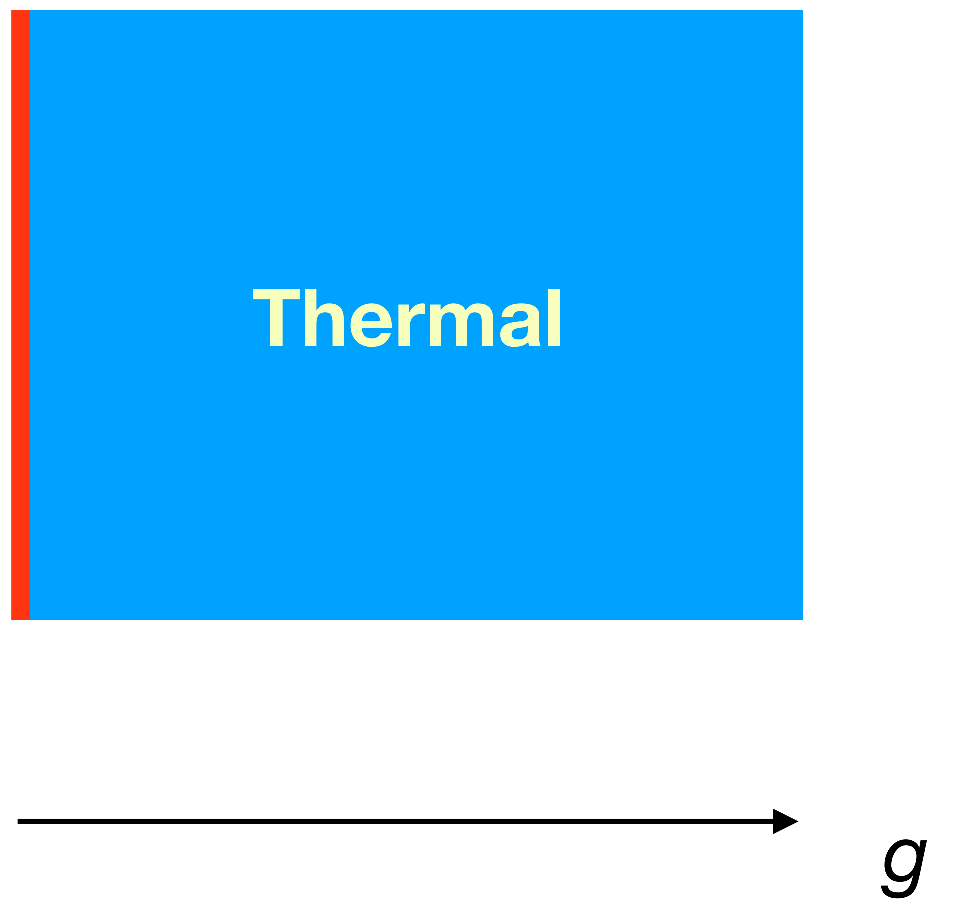
PRE (2020), PRB (2020), Ann. Phys. (2021), arXiv:2105.09336

Breakdown of ergodicity

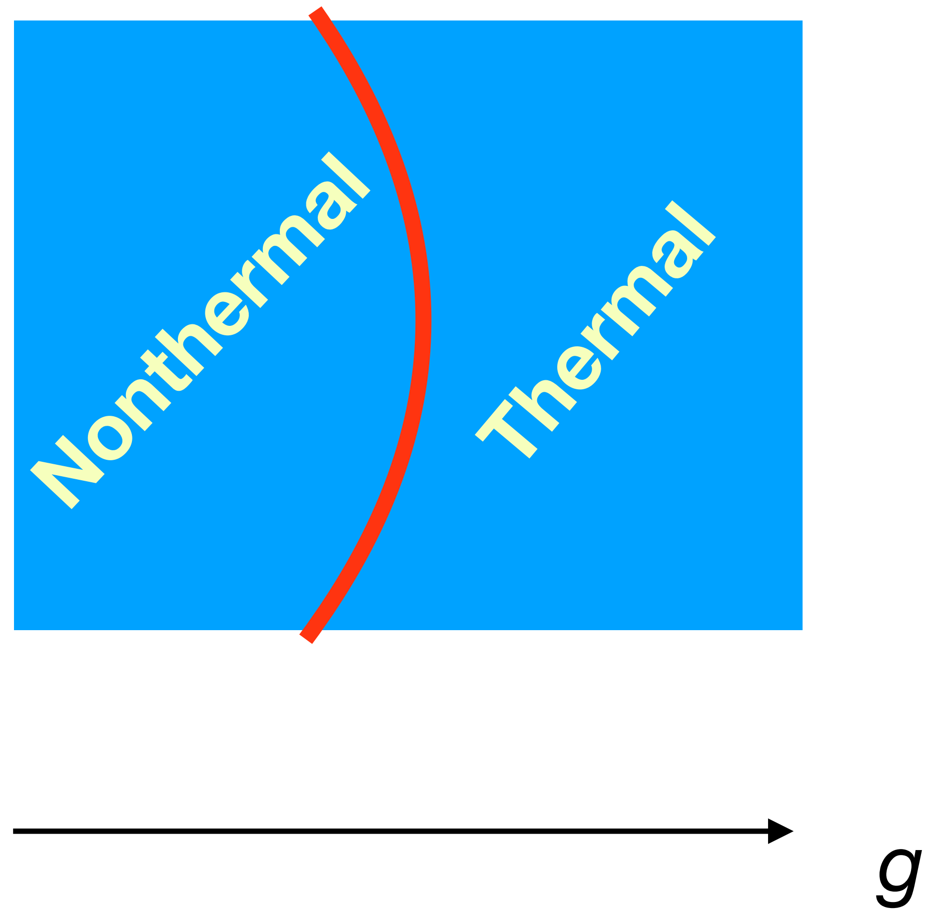
Counterexamples to thermalization

Singular point



"Generic" counterexamples ?

Phase transition:
thermalizing vs nonthermalizing phase of matter?



Open questions: Disorder induced ergodicity breakdown

- How to establish efficient measures to detect key features of ergodicity breaking transition in finite systems?

- How to explain anomalously slow dynamics in systems with disorder?

Message #1

- How to establish efficient measures to detect key features of ergodicity breaking transition in finite systems?

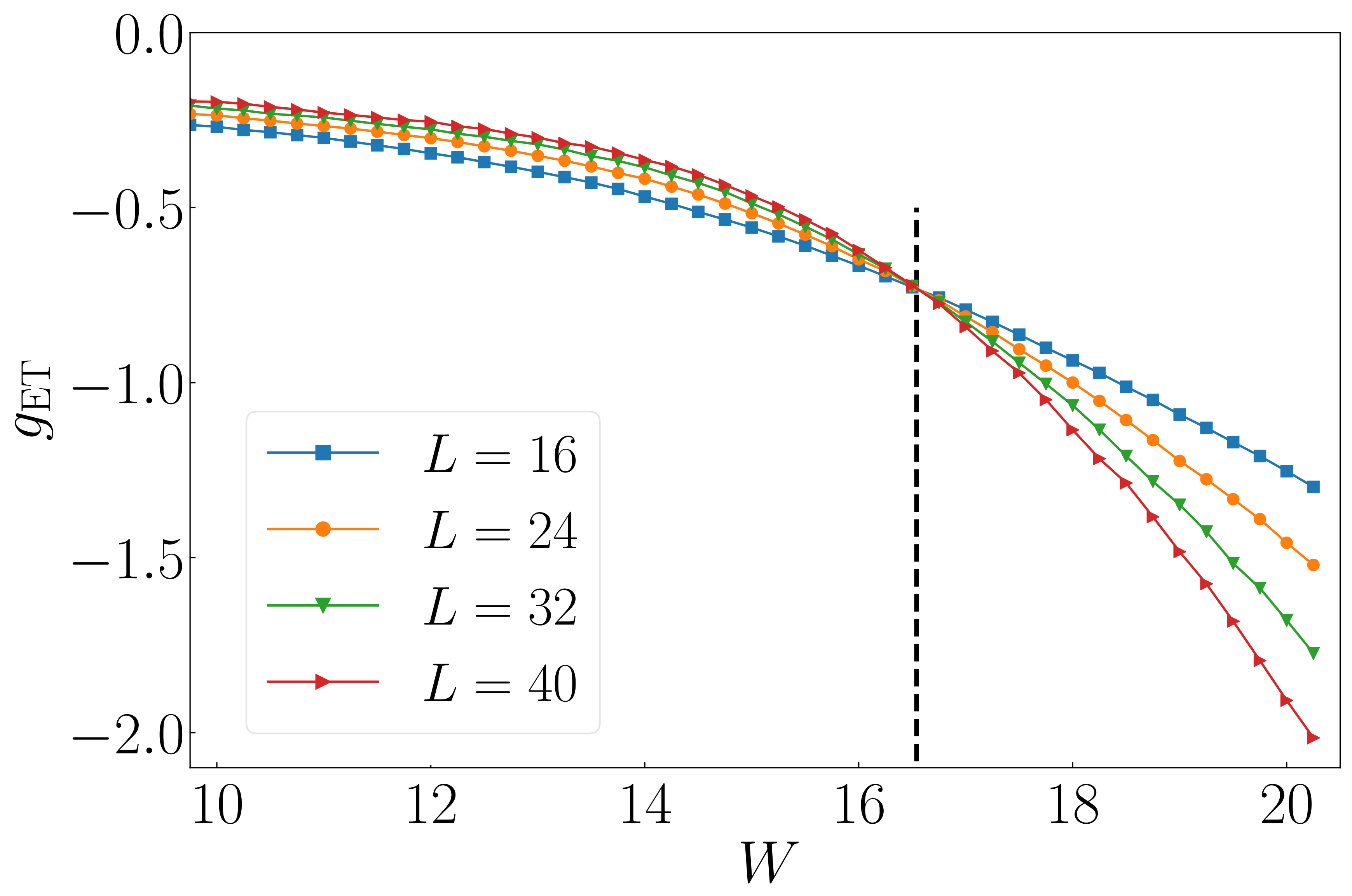
Find an appropriate reference system, for which:

- a transition to localization is well established
- the numerics is in the asymptotic regime
- allows for using identical (or analogous) measures of a transition

Reference system: Delocalization-localization transition in the **3D Anderson model**

Edwards and Thouless … 50 years later

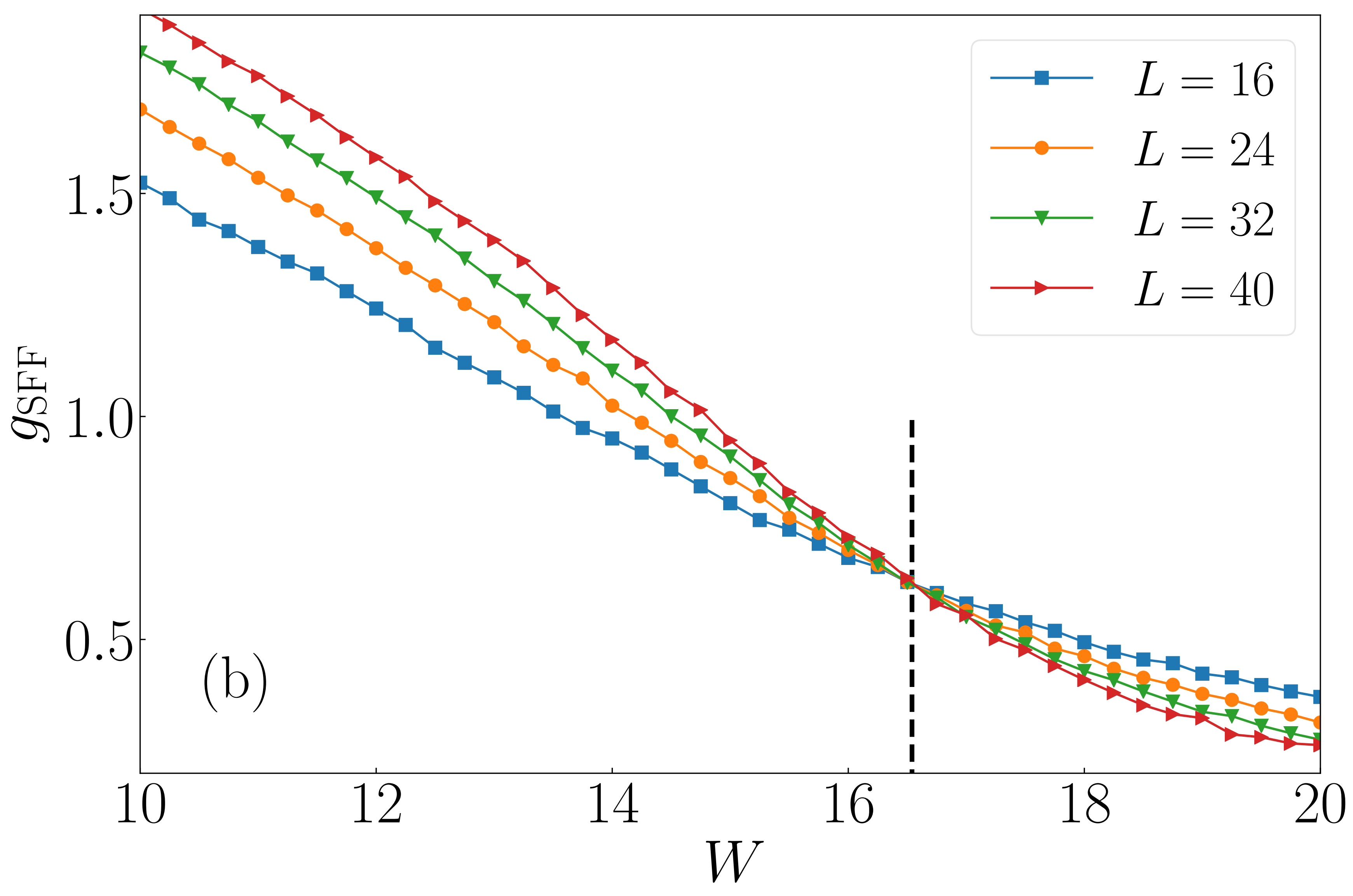
- Edwards and Thouless (1972): get E_{ET} from sensitivity to boundary conditions



$$g_{\mathrm{ET}} = \log \frac{E_{\mathrm{ET}}}{\Delta}$$

$$(E_{\mathrm{ET}})_\alpha = E_\alpha(\Phi = \pi) - E_\alpha$$

- Another perspective: get t_{Th} from the spectral form factor (SFF)



$$g_{\mathrm{SFF}} = \log \frac{t_{\mathrm{H}}}{t_{\mathrm{Th}}}$$

Šuntajs, Prosen, LV, Ann. Phys. (2021)

Edwards and Thouless … 50 years later

- Many-body problem

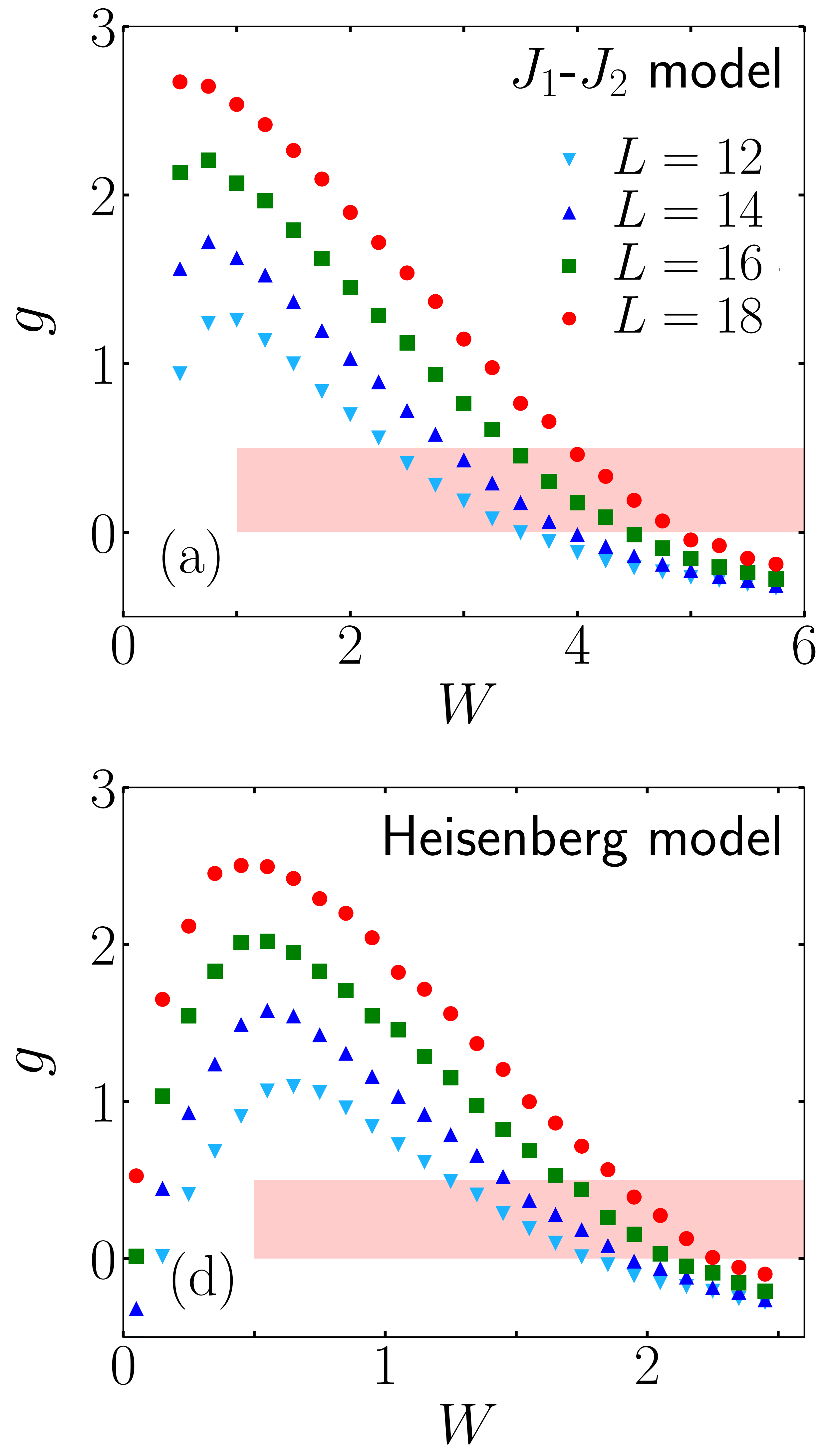
The ratio of Heisenberg time (inverse mean level spacing) and Thouless time (from the SFF) as a measure of quantum chaos

$t_{\mathrm{H}}/t_{\mathrm{Th}} \to \infty$ Emergence of quantum chaos

$t_{\mathrm{H}}/t_{\mathrm{Th}} \to 0$ Breakdown of quantum chaos

$$g = \log_{10}(t_{\mathrm{H}}/t_{\mathrm{Th}})$$

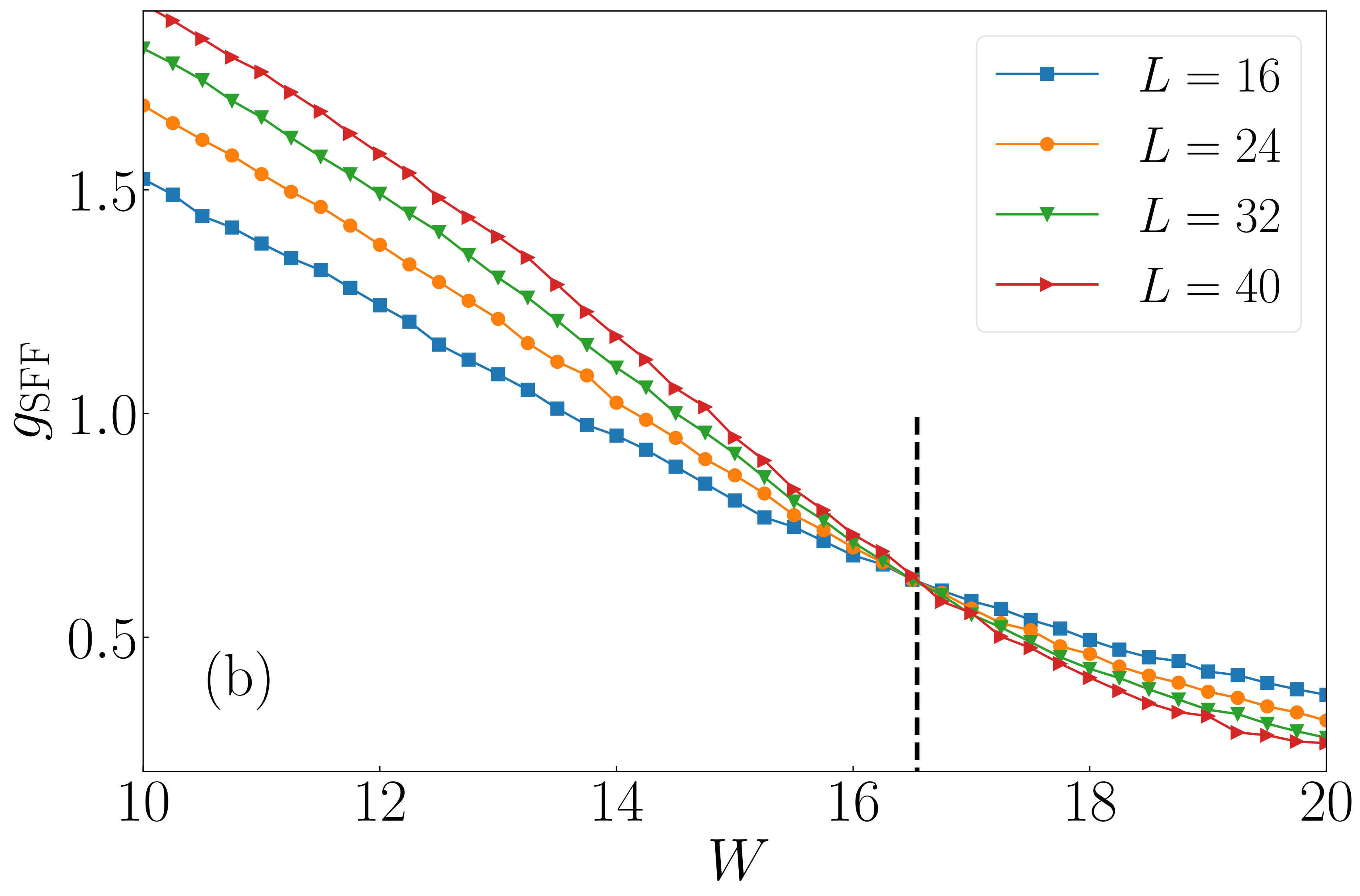
Šuntajs, Bonča, Prosen, LV, PRE (2020)



Ergodicity breaking transition: a transition at $t_{\mathrm{Th}} \approx t_{\mathrm{H}}$

$$g = \log_{10}(t_{\mathrm{H}}/t_{\mathrm{Th}})$$

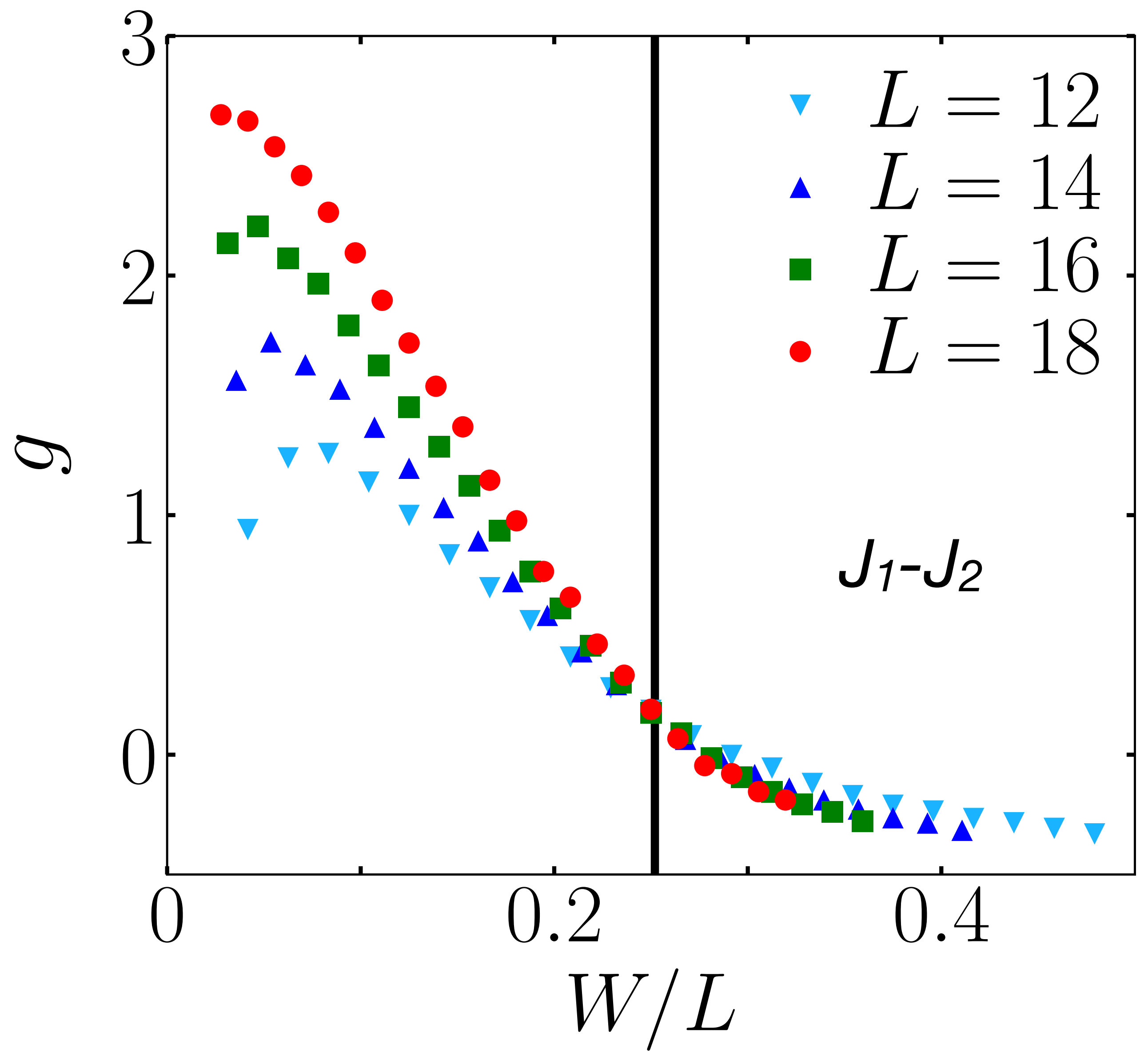
3D Anderson model versus disordered spin chains

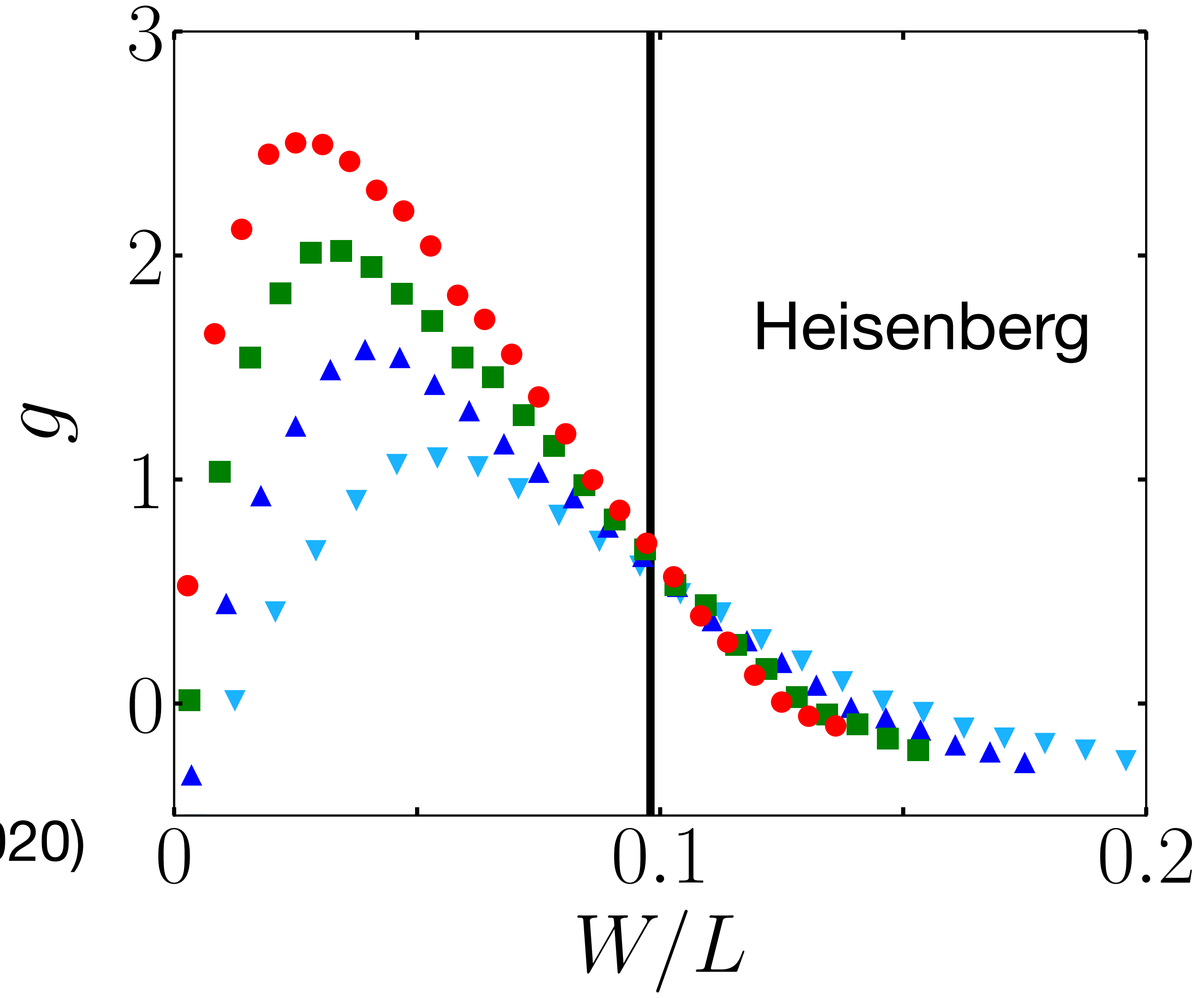


Šuntajs, Prosen, LV, Ann. Phys. (2021)

Also: Sierant, Delande, Zakrzewski, PRL 124 (2020)

$$\boxed{t_{\mathrm{H}}/t_{\mathrm{Th}} = \mathrm{const}}$$





Šuntajs, Bonča, Prosen, LV, PRE (2020)

Message #2

- How to explain anomalously slow dynamics in systems with disorder?

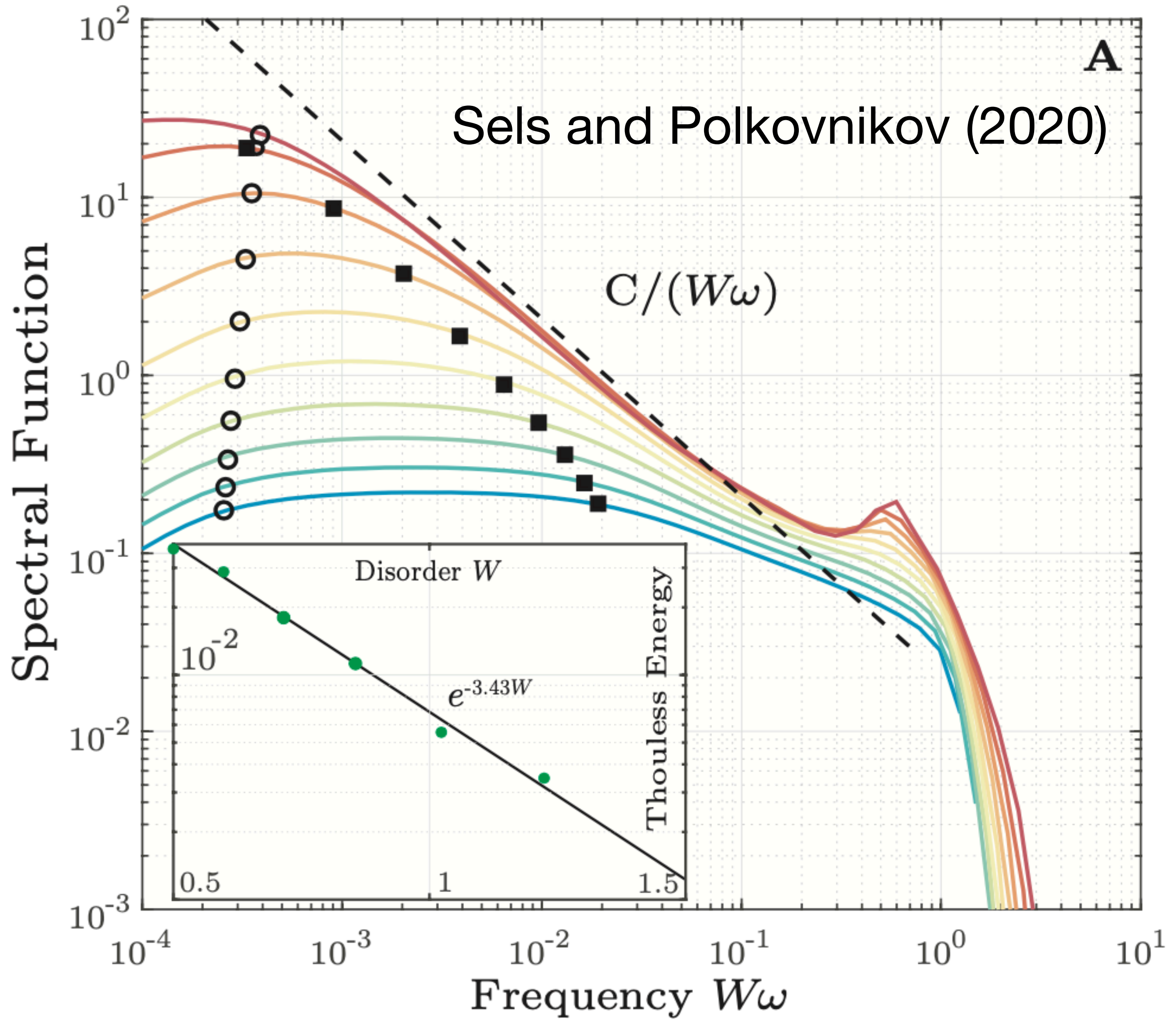
Challenges:

- Subdiffusion

Bar Lev *et al* (2015), Agarwal *et al* (2015), Luitz *et al* (2016), Khait *et al* (2016), Žnidarič *et al* (2017), …

- Scaling of the spin-1/2 S^z spectral function as $\propto 1/\omega$

Mierzejewski et al (2016), Serbyn et al (2017), Sels and Polkovnikov (2020)



Anomalous dynamics as a consequence of **proximity to an integrable point** (Anderson insulator)

Observables

- Spectral function

$$S(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}t\ e^{i\omega t - |t|0^+}\langle e^{i\hat{H}t}\hat{A}e^{-i\hat{H}t}\hat{A}\rangle$$

- Integrated spectral function

$$I(\omega) = \int_{-\omega}^{\omega} \mathrm{d}\omega' S(\omega') = \frac{1}{\mathscr{D}}\sum_{m,n=1}^{\mathscr{D}} \theta\left(\omega - |E_m - E_n|\right) A_{mn}^2$$

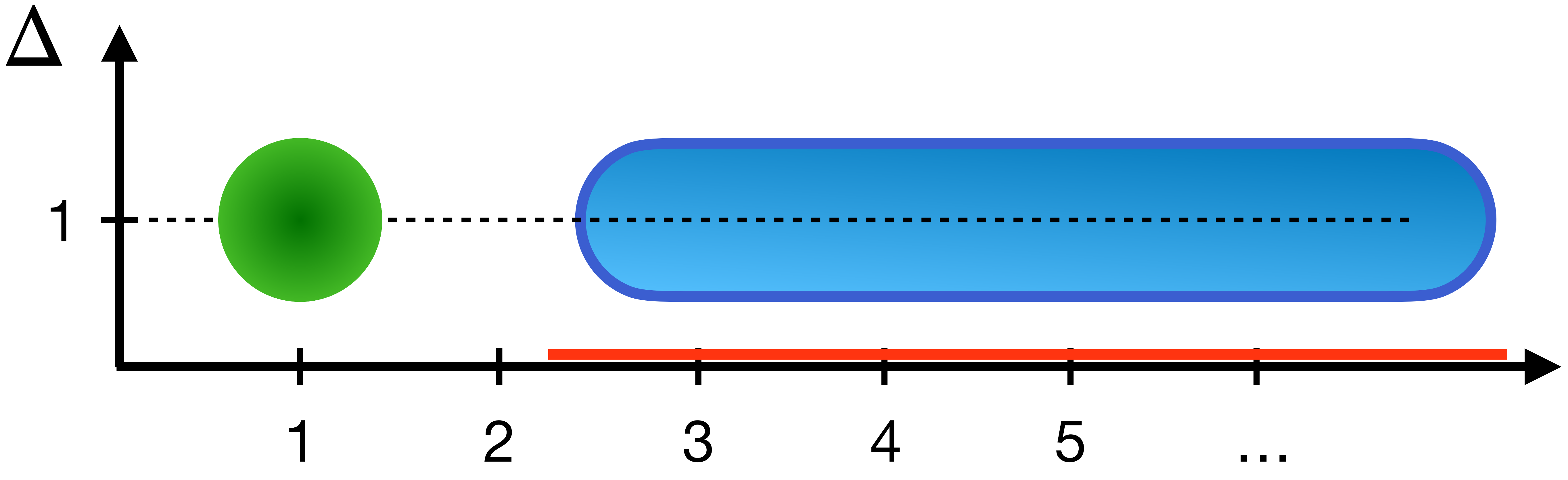
$$A_{mn} \equiv \langle m|\hat{A}|n\rangle$$

- Regular part of the integrated spectral function

$$\tilde{I}(\omega) = I(\omega) - \frac{1}{\mathscr{D}}\sum_{n=1}^{\mathscr{D}} A_{nn}^2$$

Random field XXZ Heisenberg chain

$$\hat{H} = J \sum_i (\hat{S}_i^x \hat{S}_{i+1}^x + \hat{S}_i^y \hat{S}_{i+1}^y + \Delta \hat{S}_i^z \hat{S}_{i+1}^z) + \sum_i h_i \hat{S}_i^z \qquad h_i \in [-W, W]$$



Integrable system

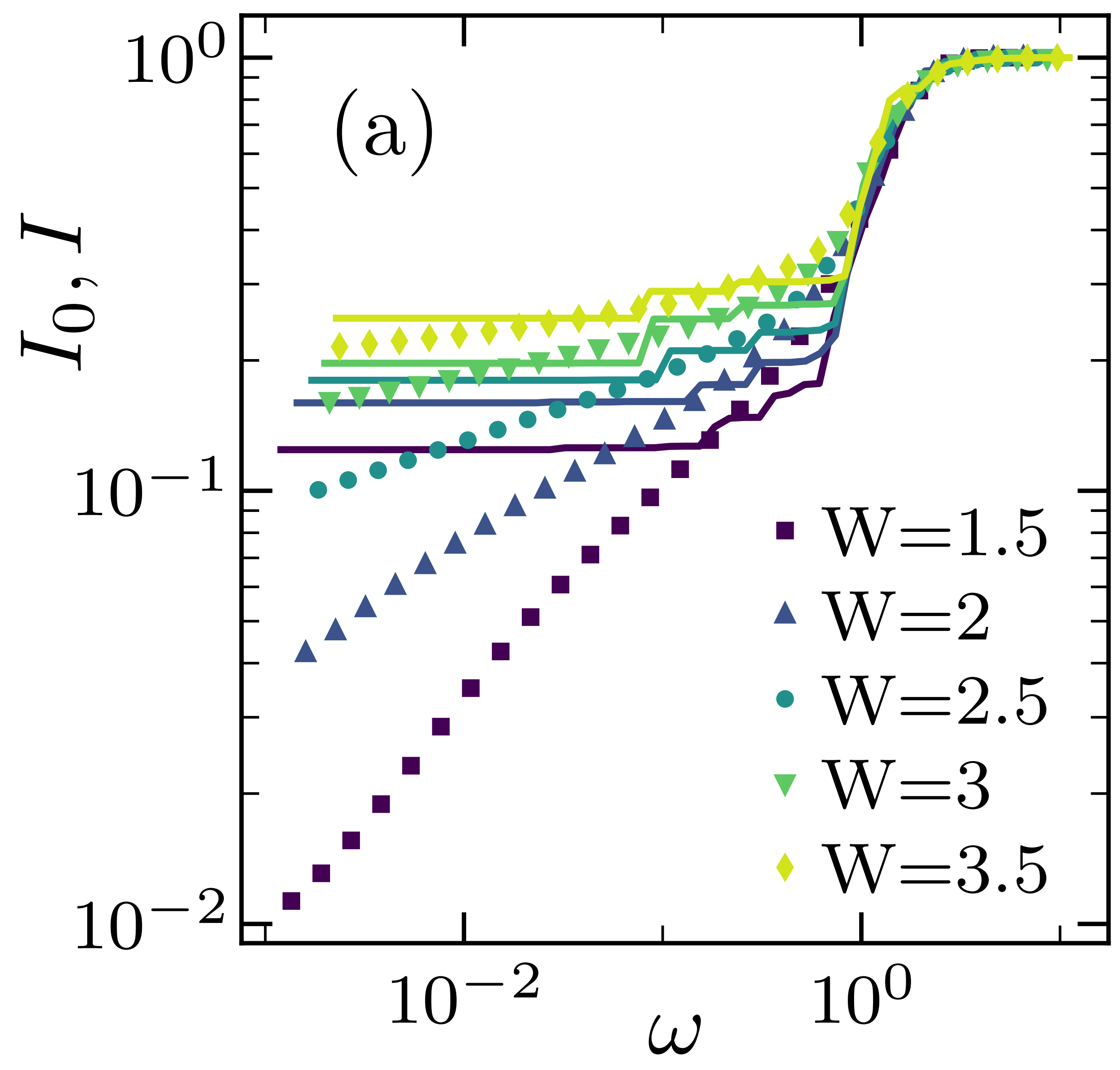
Anderson insulator

Anderson LIOMs $\{\hat{Q}_\alpha\}$

(LIOM = local integral of motion)

Integrated spectral functions at $\Delta = 0$ and $\Delta = 1$ for spin imbalance

$$\hat{A} = \frac{2}{\sqrt{L}} \sum_i (-1)^i \hat{S}_i^z$$



Anderson insulator ($\Delta = 0$, lines)

- $I_0(\omega \ll J) \simeq \text{const}$
- $S_0(\omega \ll J) \simeq D_0 \delta(\omega)$
- $S_{\mathrm{M},0}(\omega) = \sum_\alpha D_\alpha \delta(\omega)\,, \quad D_\alpha = \frac{\langle \hat{A} \hat{Q}_\alpha \rangle^2}{\langle \hat{Q}_\alpha \hat{Q}_\alpha \rangle}$

D_α follows from Mazur bound (as in Mierzejewski and LV, 2020)

Interacting system ($\Delta = 1$, symbols)

$S(\omega \ll J) = ?$

"Anomalous dynamics"

↓

Phenomenological model

Phenomenological description of interacting systems

- Step 1: **Delocalization of Anderson LIOMs**

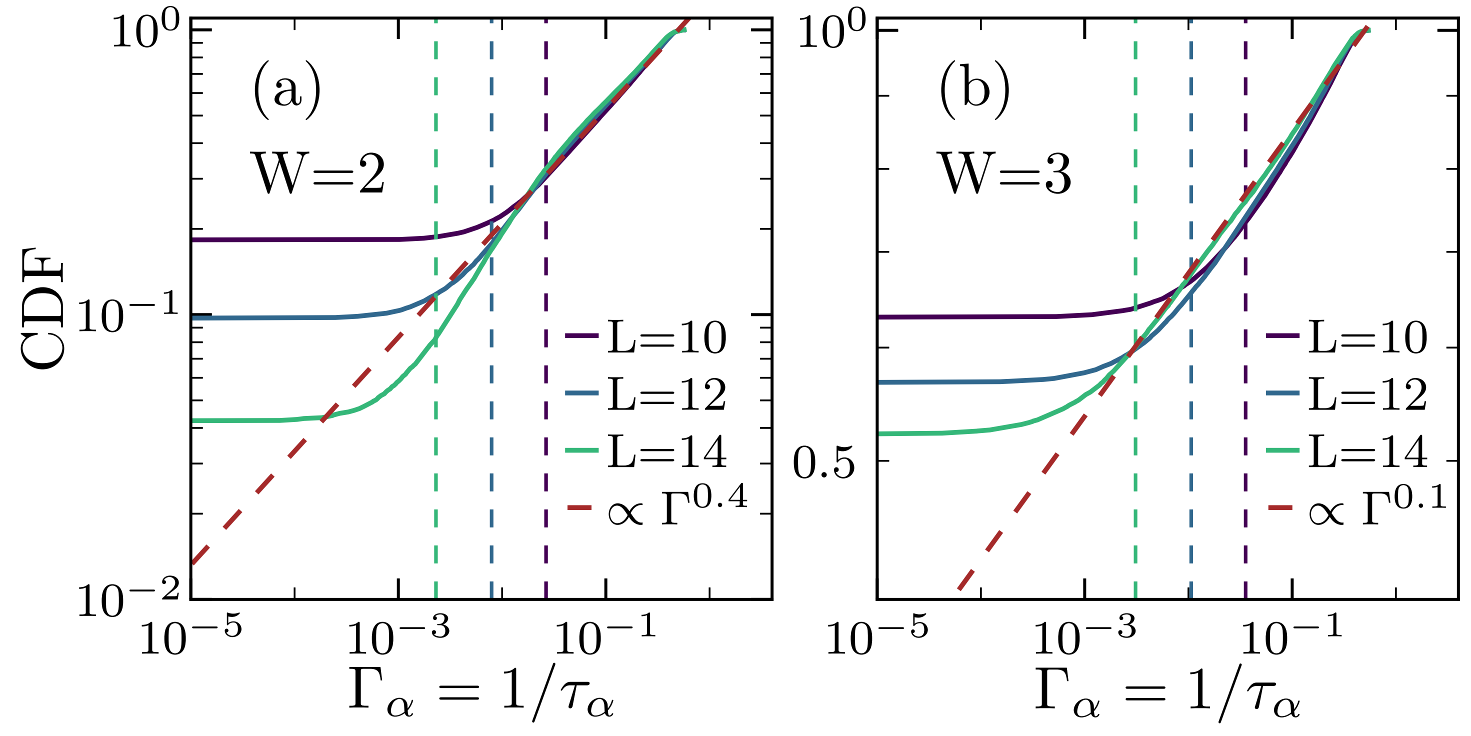
$$\langle \hat{Q}_\alpha(t)\hat{Q}_\alpha\rangle \propto \exp(-t/\tau_\alpha)$$

$$S_{\mathrm{M},0}(\omega) = \sum_\alpha D_\alpha \delta(\omega)\,, \quad D_\alpha = \frac{\langle \hat{A}\hat{Q}_\alpha\rangle^2}{\langle \hat{Q}_\alpha \hat{Q}_\alpha\rangle} \longrightarrow S_\mathrm{M}(\omega \ll J) = \sum_{\alpha=1}^{N} D_\alpha \frac{1}{\pi}\frac{\tau_\alpha}{(\omega\tau_\alpha)^2+1}$$

$$\Delta = 0 \qquad\qquad \Delta = 1$$

- Step 2: **Power-law distribution of relaxation times**

$$f_\tau(\tau) \propto 1/\tau^\mu \qquad \tau \in [\tau_\mathrm{min}, \tau_\mathrm{max}]$$

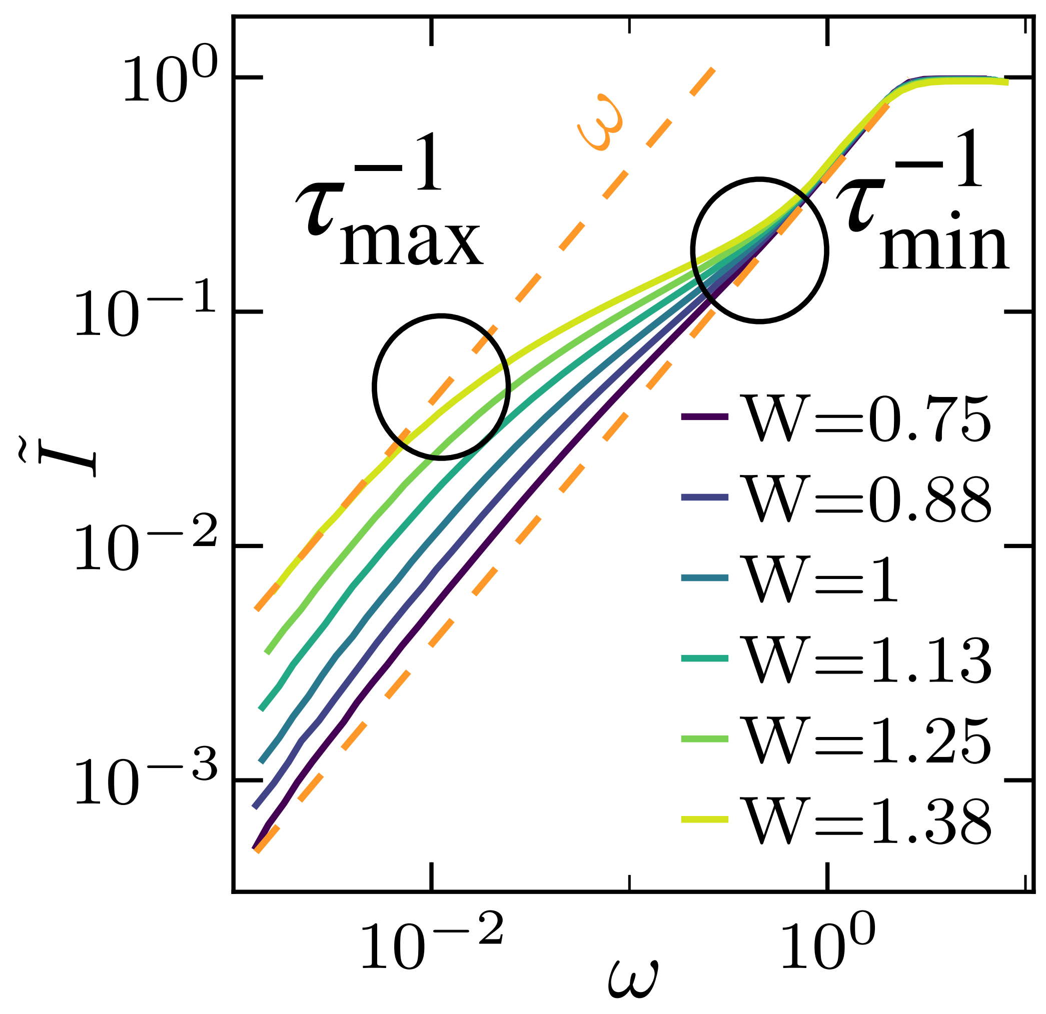


$$\boxed{S_\mathrm{M}(\omega) = \frac{\bar{D}_0}{\pi}\int_{\tau_\mathrm{min}}^{\tau_\mathrm{max}} \frac{d\tau}{\tau^{\mu-1}}\frac{1}{(\omega\tau)^2+1}}$$

General properties of the spectral described by $S_{\mathrm{M}}(\omega) = \frac{\bar{D}_0}{\pi} \int_{\tau_{\min}}^{\tau_{\max}} \frac{d\tau}{\tau^{\mu-1}} \frac{1}{(\omega\tau)^2 + 1}$

- Relevant when $\tau_{\max} \gg \tau_{\min}$

  Not the case at $\Delta \sim W \sim 1$

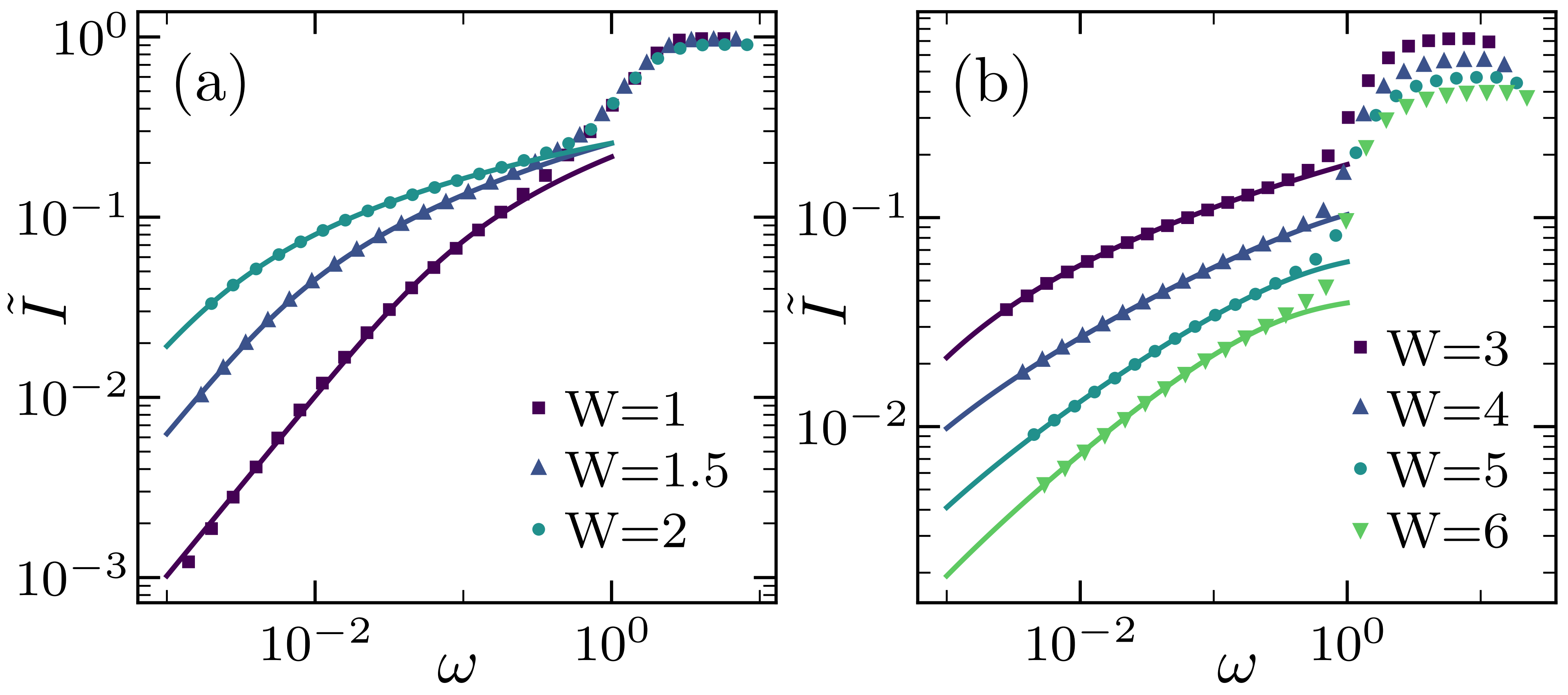


- Case $\mu = 1$

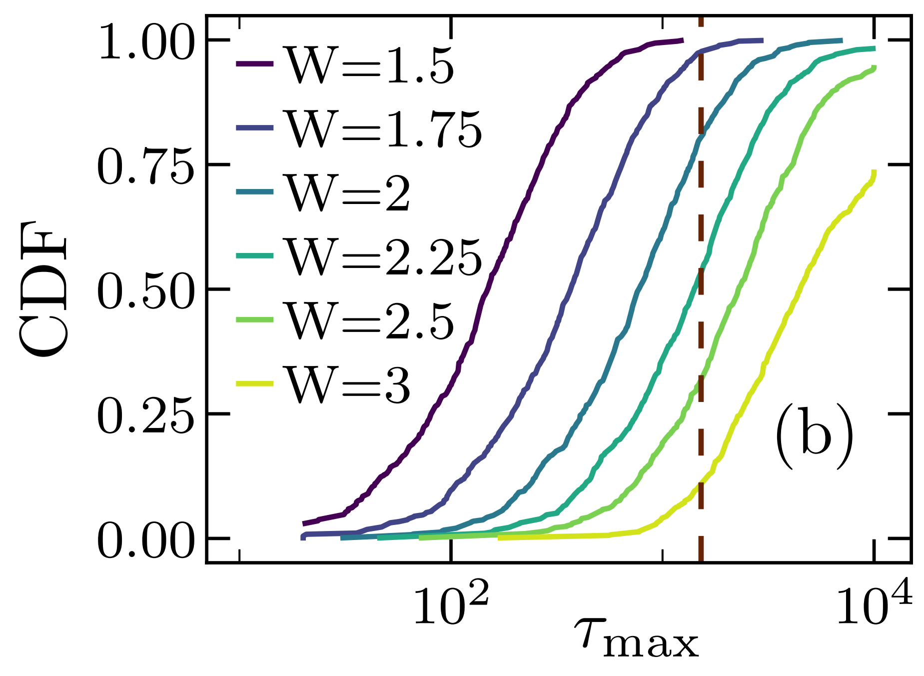
$$S_{\mathrm{M}}(\omega) = \frac{\bar{D}_0}{\pi} \frac{\arctan(\omega\tau_{\max}) - \arctan(\omega\tau_{\min})}{\omega}$$

More generally, $S_{\mathrm{M}}(\omega) \propto 1/\omega^{\eta}$ with $\eta \simeq 2 - \mu$ (at $\mu < 2$)

Numerical tests for disorder-averaged results



$$S_{\mathrm{M}}(\omega) = \frac{\bar{D}_0}{\pi} \int_{\tau_{\min}}^{\tau_{\max}} \frac{d\tau}{\tau^{\mu-1}} \frac{1}{(\omega\tau)^2 + 1}$$

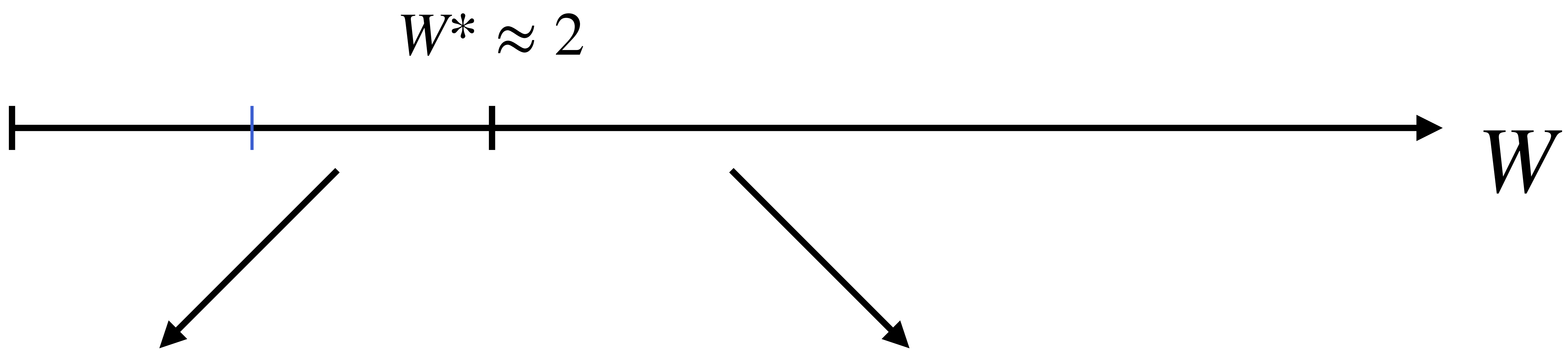


$\tau_{\max} \approx t_{\mathrm{H}}$ at $W = W^* \approx 2$

Interpretation

- Anomalous dynamics: It arises from certain Anderson LIOMs that acquire **finite relaxation times**

- **Distribution of relaxation times**

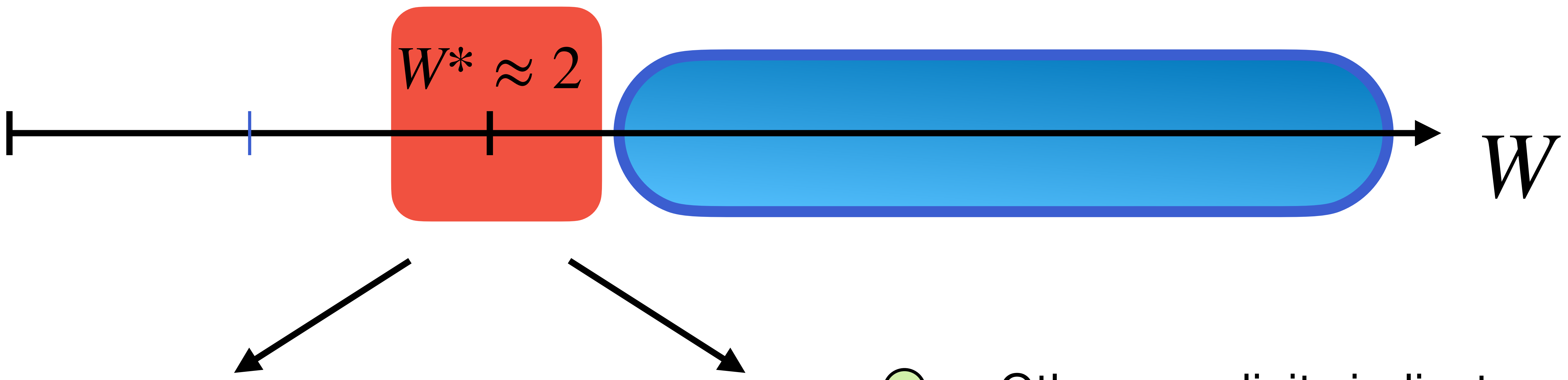


Delocalized Anderson LIOMs

Coexistence of two types of LIOMs

- those for which $\tau_\alpha > t_\mathrm{H}$ (they appear to be exactly conserved)
- those for which $\tau_\alpha < t_\mathrm{H}$ (not conserved due to interactions)

Conclusions



- Spectral functions

Certain Anderson LIOMs relax on time scales $\tau \gtrsim t_{\mathrm{H}}$

- Other ergodicity indicators

$t_{\mathrm{Th}} \approx t_{\mathrm{H}}$ — Šuntajs *et al* (2020)

r and entanglement entropy depart from RMT predictions

Fidelity susceptibility is maximal — Sels and Polkovnikov (2020)

Opening of the Schmidt gap — Grey *et al* (2018)

… and the gap in the spectrum of the eigenstate one-body density matrix

Bera *et al* (2015)

Correlation-hole time in survival probability reaches t_{H}

Schiulaz *et al* (2019)

Thank you!