The resonance model of MBL-thermal finite size crossover

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Many body localisation: the good

$$H = \sum_i \left(\vec{\sigma}_i \cdot \vec{\sigma}_{i+1} + h_i \sigma_i^z \right) \qquad h_i \in [-W, W] \qquad T = \infty, \qquad W > W_\mathrm{c}$$

Gornyi, Mirlin & Polyakov (2005), Basko Aleiner & Altshuler (2006), Oganesyan & Huse (2007, 2014), Pal & Huse (2010), …

Experiments: Bordia et al, Schreiber et al, Lukin et al, Choi et al, Xu et al, Roushan et al, Smith et al, Luschen et al, Rispoli et al, Guo et al…

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- Infinite time memory / no transport ✓
- Exponentially localised integrals of motion "L-bits" $[H, \tau_i^z] = 0$ ✓
- Logarithmically growing entropy $S \sim \log t$ ✓
- Neighbouring sub-diffusive Griffiths phase $\langle \sigma_i^z(t) \sigma_i^z(0) \rangle \sim t^{-1/z}, \quad z > 2$ ✓

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Many body localisation: the problems

- Correlation length exponent $\nu = \infty$ ✗ $\nu \approx 1 \leq 2$, strong drift
- Broadly distributed t_{th} in sub-diffusive phase $p(t_{\mathrm{th}}) \sim t_{\mathrm{th}}^{-\mu}$ ✗ $p(t_{\mathrm{th}}) \sim \mathrm{e}^{-c(t_{\mathrm{th}}-\bar{t})^{\alpha}}$
- Avalanches ✗
- Diverging thermalisation times $t_{\mathrm{th}}(W \to W_c) \to \infty$ ✗ $t_{\mathrm{th}} \sim \mathrm{e}^{cW}$

Kjäll et al (2014), Luitz et al (2015), Schulz et al (2020), Schiulaz et al (2019), Šuntajs et al (2019), Sels and Polkovnikov (2020)

Many body localisation: the problems

Are these inconsistencies incompatible with MBL?

Many body localisation: the problems

Are these inconsistencies incompatible with MBL?

No. They are natural consequences of the small accessible system sizes.

Main message

<u>Assumptions</u>

- MBL
- Small systems
- No rare regions

<u>Resonance Model</u>

- Model for eigenstates in MBL phase
- Pre-asymptotic scaling theory for the MBL-thermal crossover

<u>Predictions</u>

- Phenomenology of numerical crossover
- Self consistently stable MBL

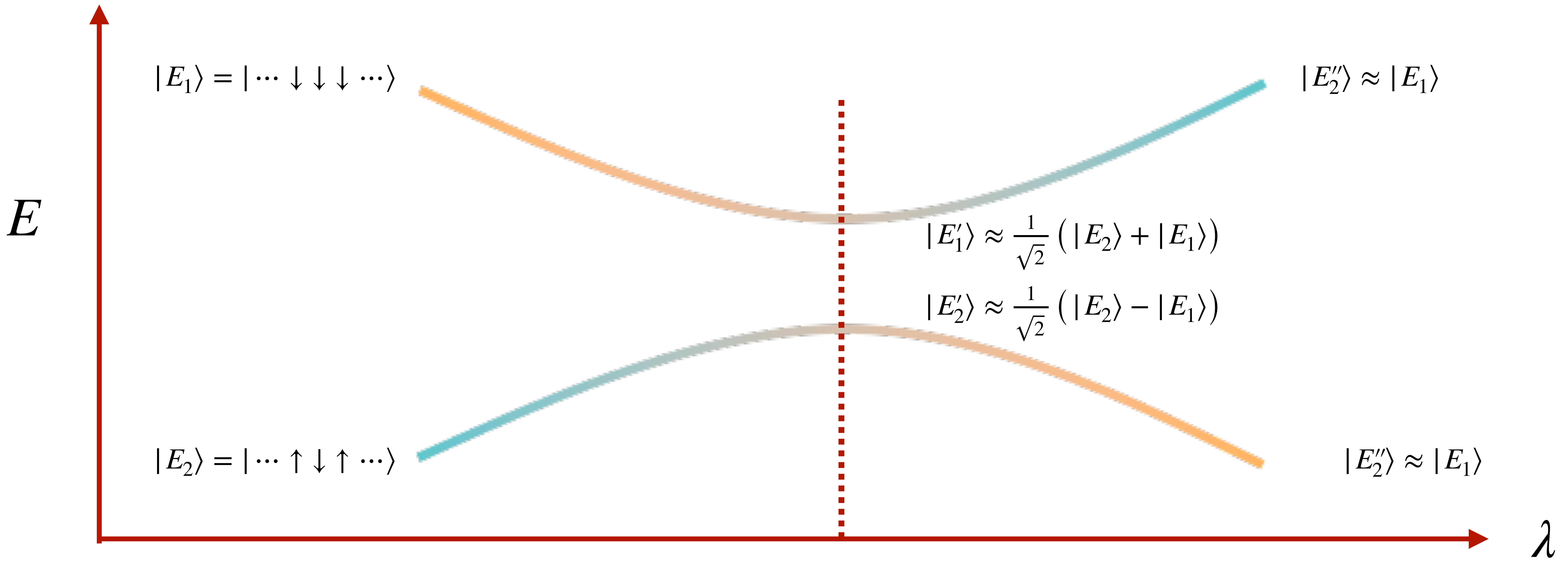
Model for eigenstates: l-bits + resonances

$$H_{\text{MBL}} = \sum_i h_i \tau_i^z + \sum_{ij} J_{ij} \tau_i^z \tau_j^z + \sum_{ijk} K_{ijk} \tau_i^z \tau_j^z \tau_k^z + \ldots$$

$$|E_a\rangle = |\cdots \uparrow \downarrow \uparrow \uparrow \uparrow \downarrow \uparrow \downarrow \uparrow \cdots\rangle$$

$$H_{\text{MBL}} \to H'_{\text{MBL}} = H_{\text{MBL}} + \lambda V$$

Model for eigenstates: l-bits + resonances



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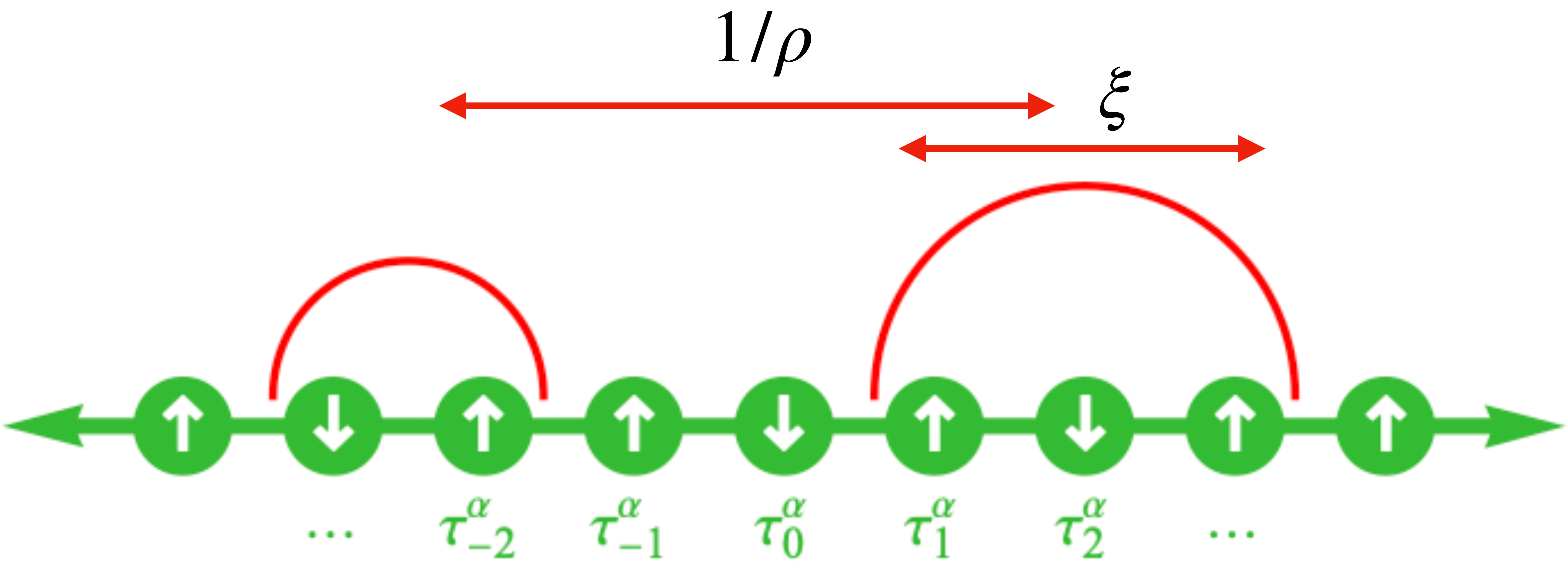
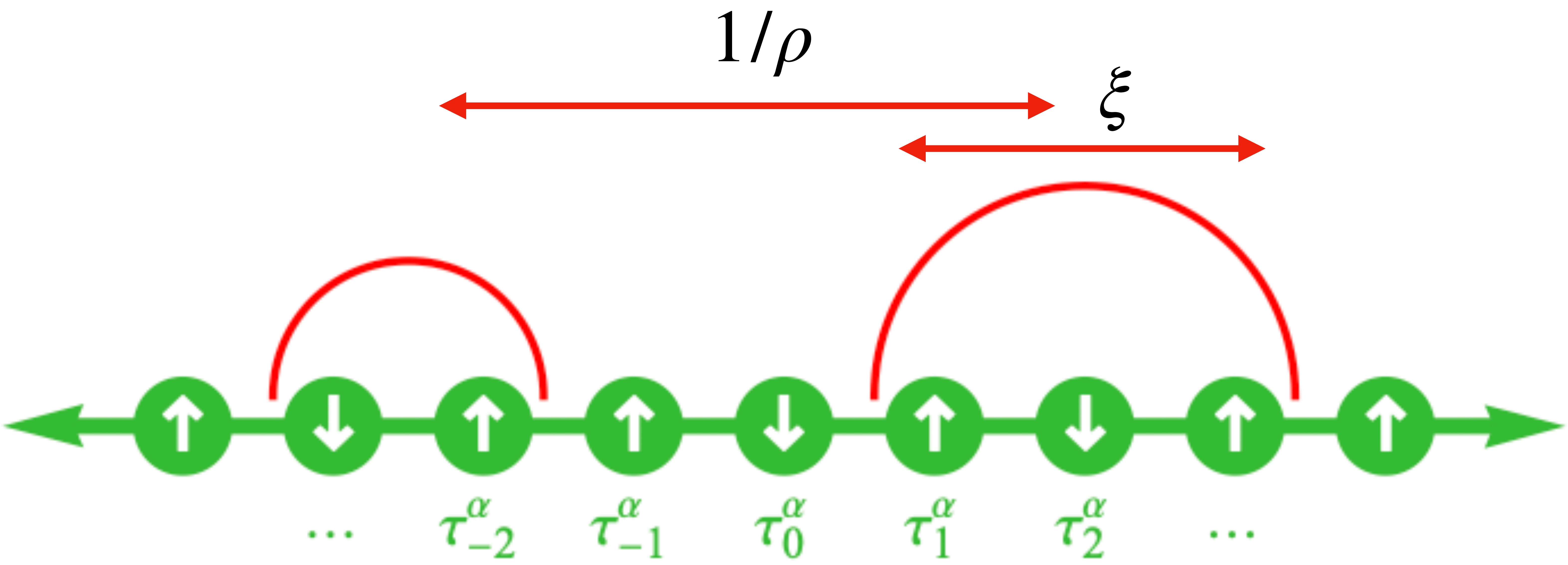


$$|E_a\rangle \propto |\cdots\uparrow\rangle \otimes \left(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle\right) \otimes |\uparrow\downarrow\rangle \otimes \left(|\downarrow\uparrow\downarrow\rangle + |\uparrow\uparrow\uparrow\rangle\right) \otimes |\uparrow\cdots\rangle$$

$$1/\rho \gg \xi$$

Model for eigenstates: l-bits + resonances



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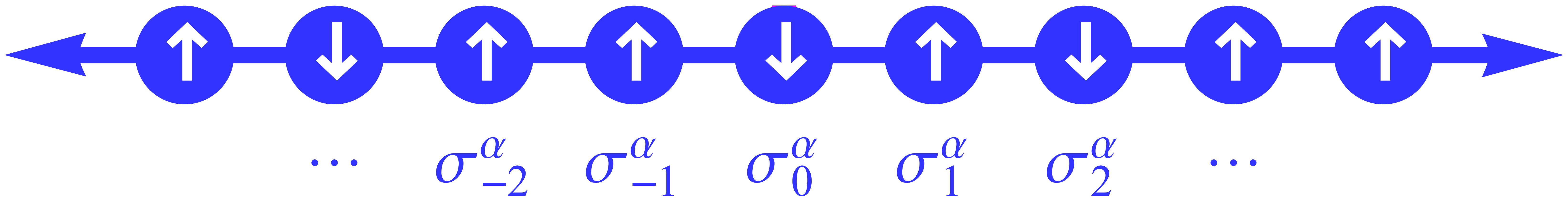
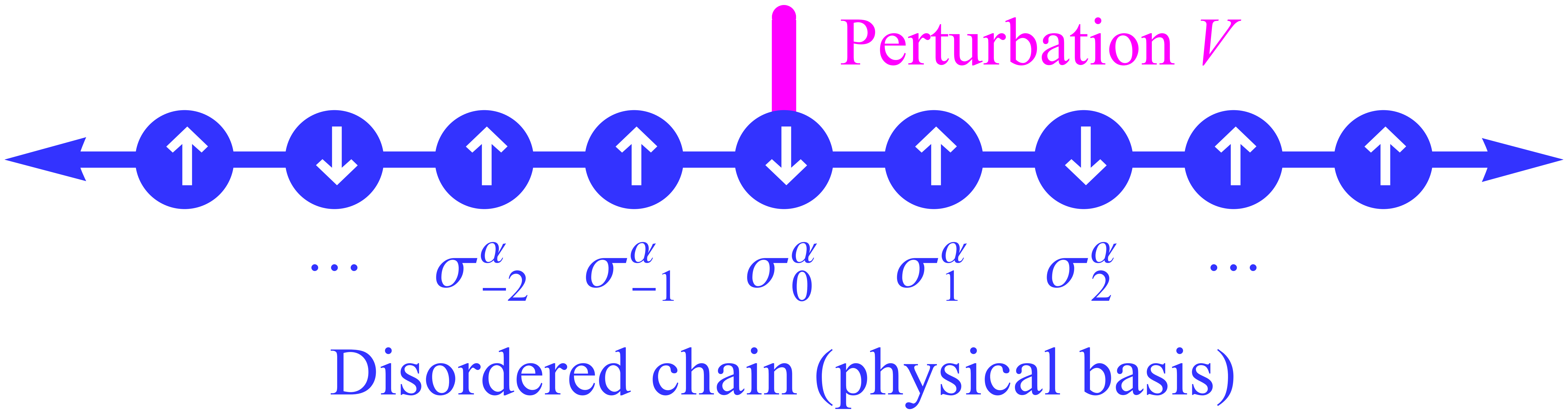
Small systems $1/\rho \gg L$ $\implies$ MBL stable for $L < \xi$ $\implies$ “scaling” with parameter L/ξ

Model for eigenstates: l-bits + resonances

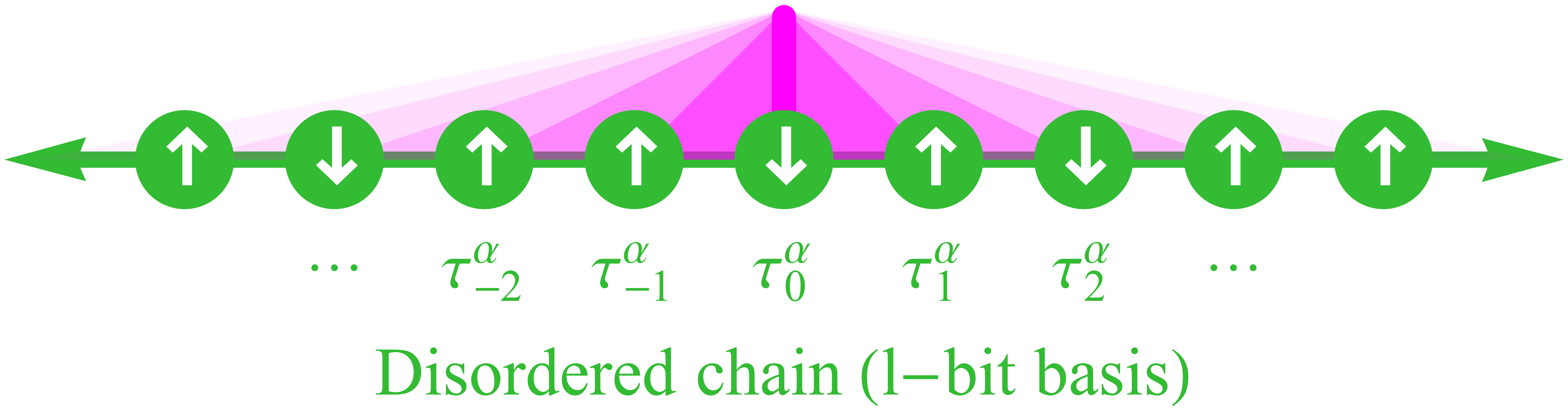


Disordered chain (physical basis)

Model for eigenstates: l-bits + resonances



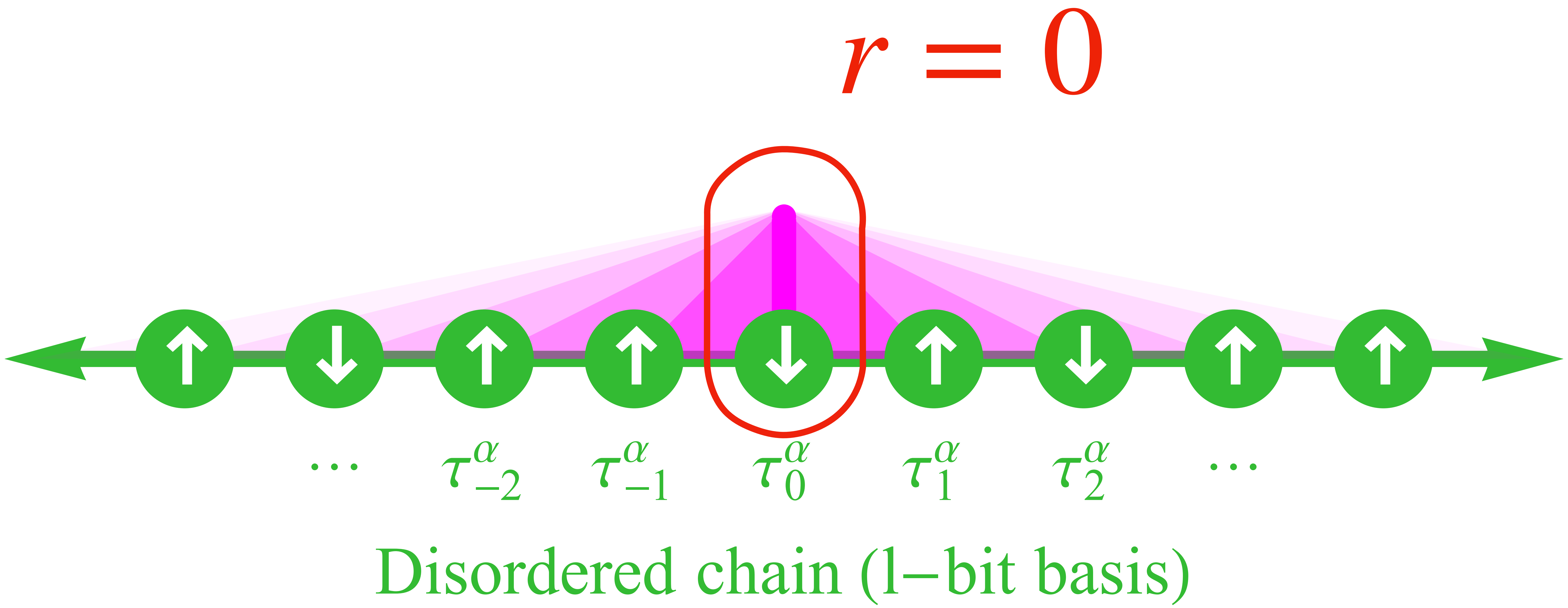
Model for eigenstates: l-bits + resonances



$$V = \sum_r V_r \qquad |V_r| \sim \mathrm{e}^{-r/\zeta}$$

decomposition of coupling

Model for eigenstates: l-bits + resonances



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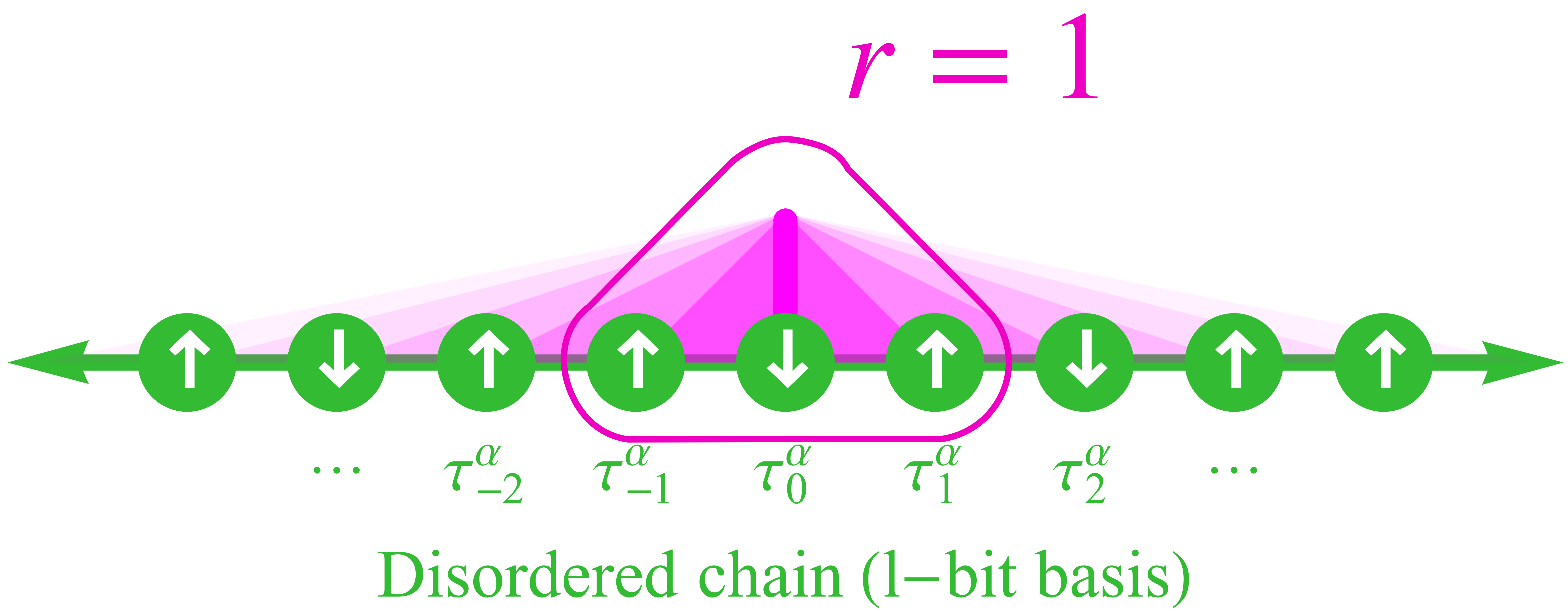

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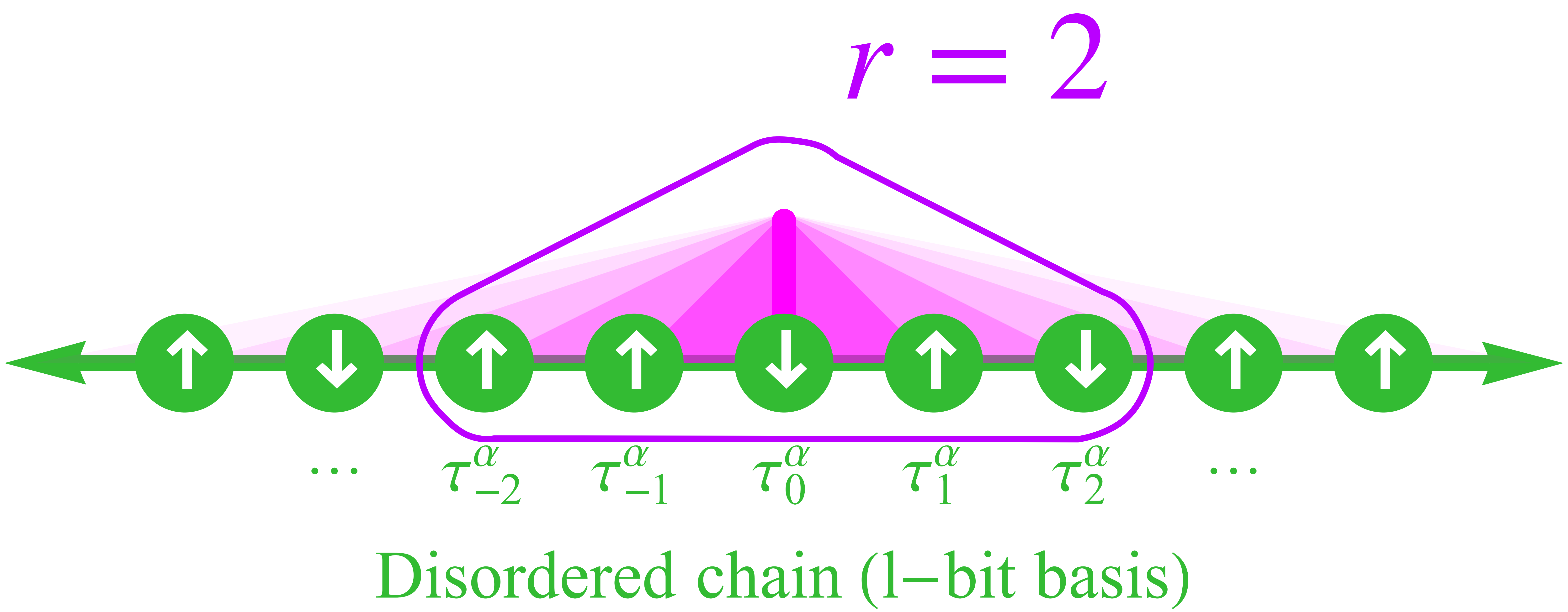

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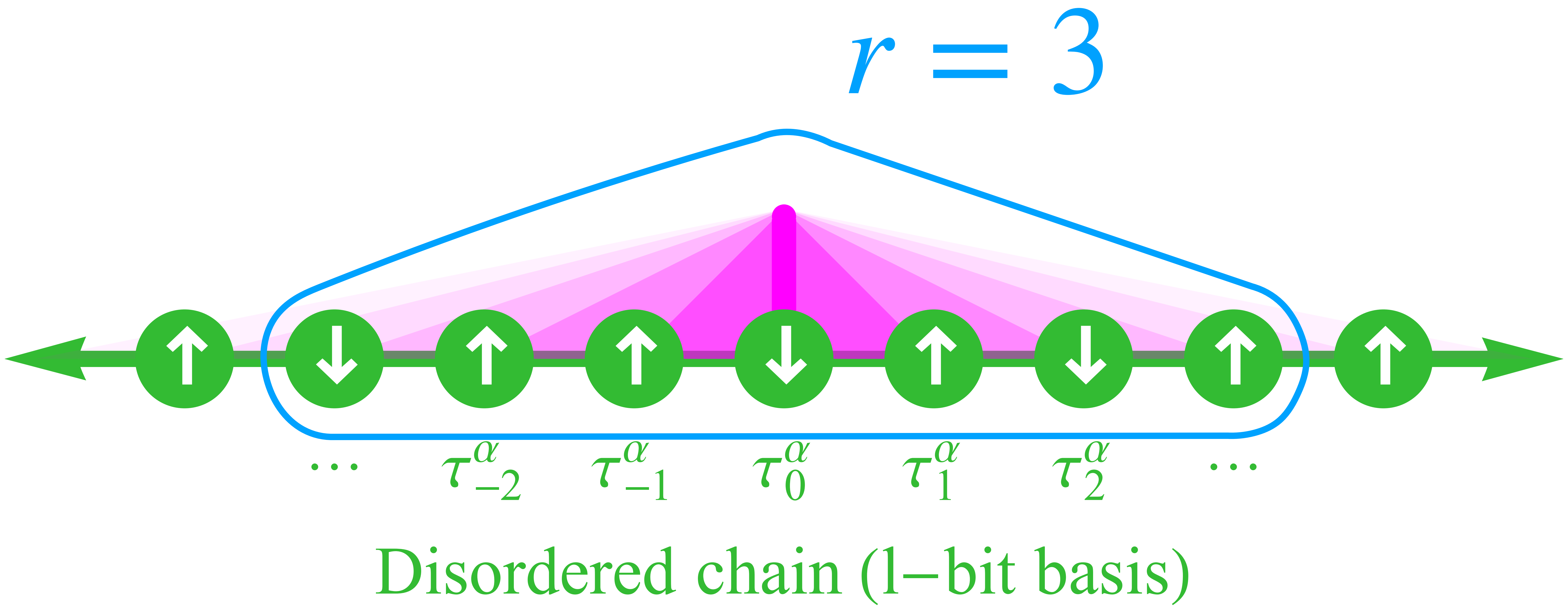


$$V = \sum_r V_r \qquad |V_r| \sim e^{-r/\zeta}$$

decomposition of coupling

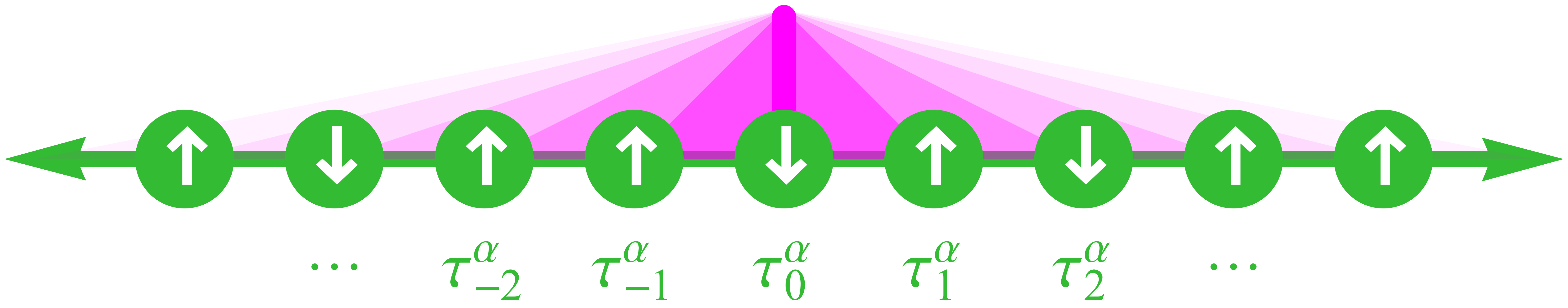
Model for eigenstates: l-bits + resonances

$$\rho(r) \sim 2^{2r+1}$$

density of states

$$\langle E_a | V_r | E_b \rangle \sim v(r) \sim \frac{e^{-r/\zeta}}{2^r}$$

matrix element scale



Disordered chain (l–bit basis)

$$V = \sum_r V_r \qquad |V_r| \sim e^{-r/\zeta}$$

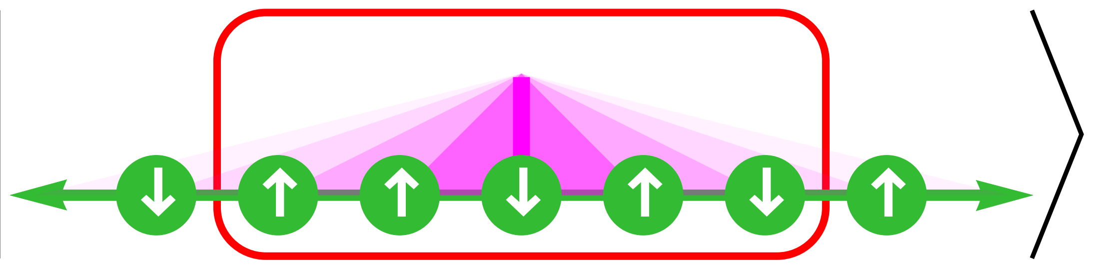
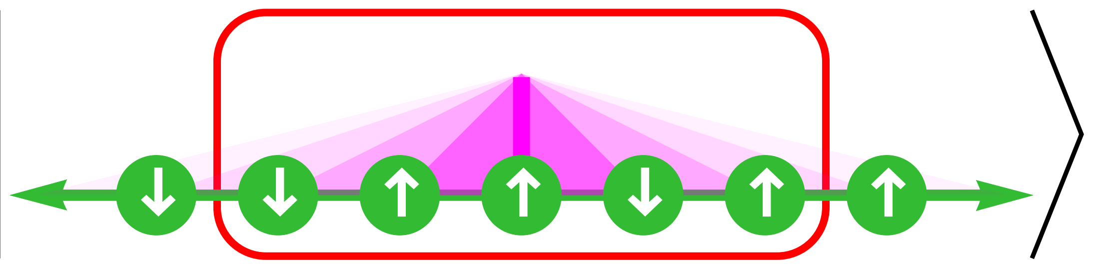
decomposition of coupling

Model for eigenstates: l-bits + resonances

$$v(r) > |E_a - E_b|$$

matrix element   level spacing

Form a resonance

$|E_a\rangle = \alpha \,|$  $\rangle \pm \beta \,|$  $\rangle$

associated timescale: $t \sim v(r)^{-1}$

$$\langle E_a | \sigma_z(t)\sigma_z(0) | E_a \rangle \approx \cos(v(r)t)$$

Pre-asymptotic scaling theory

$q(r)$ = probability that a state a resonance at range r

$$q(r) = v(r)\rho(r) = \frac{e^{-r/\xi}}{\lambda}$$

Matrix elements

Density of states

$$\xi \sim (1/\zeta - \log 2)^{-1} \quad \nu = 1$$

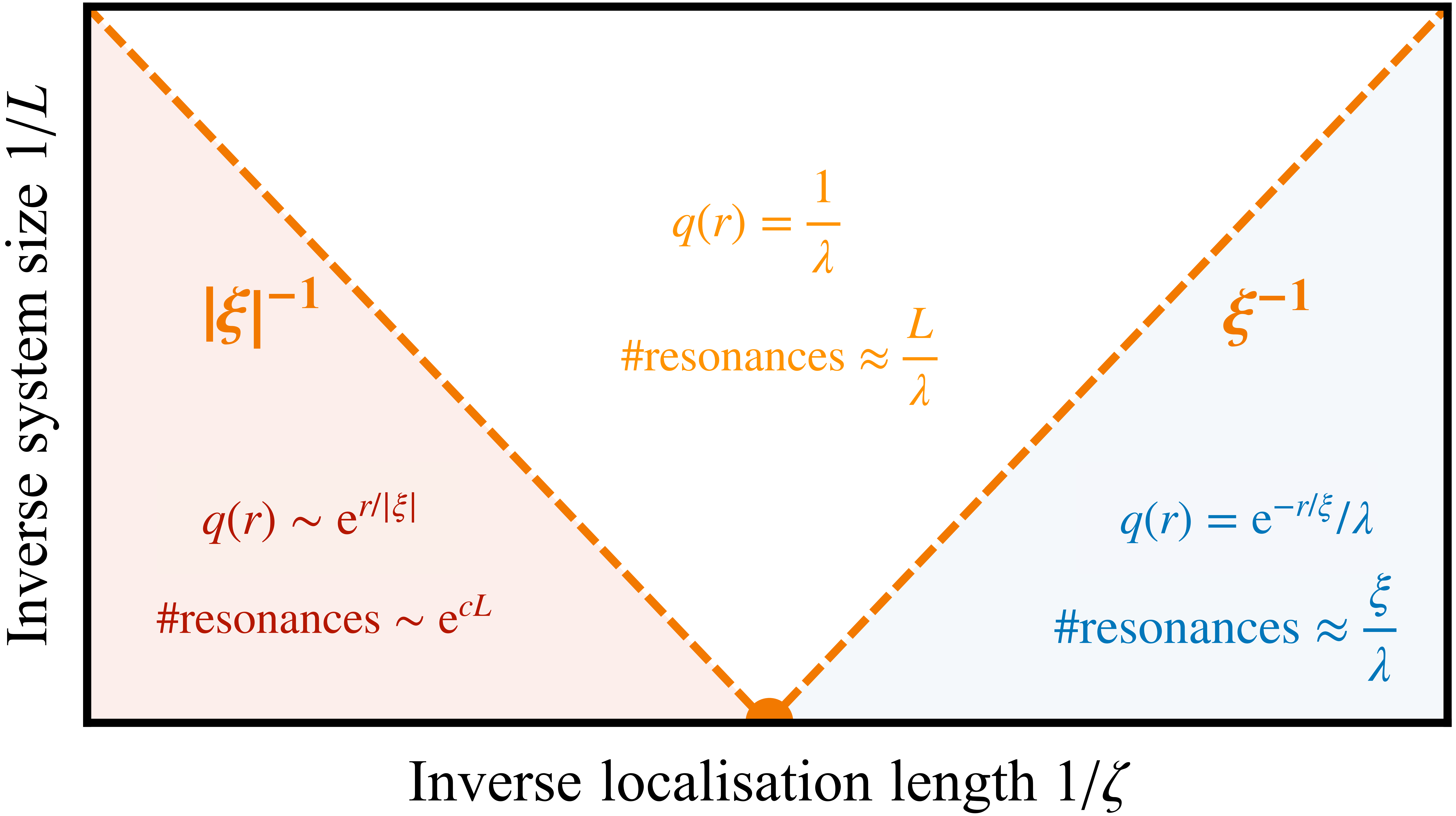
Correlation length exponent

$$\lambda(W) \approx 50$$

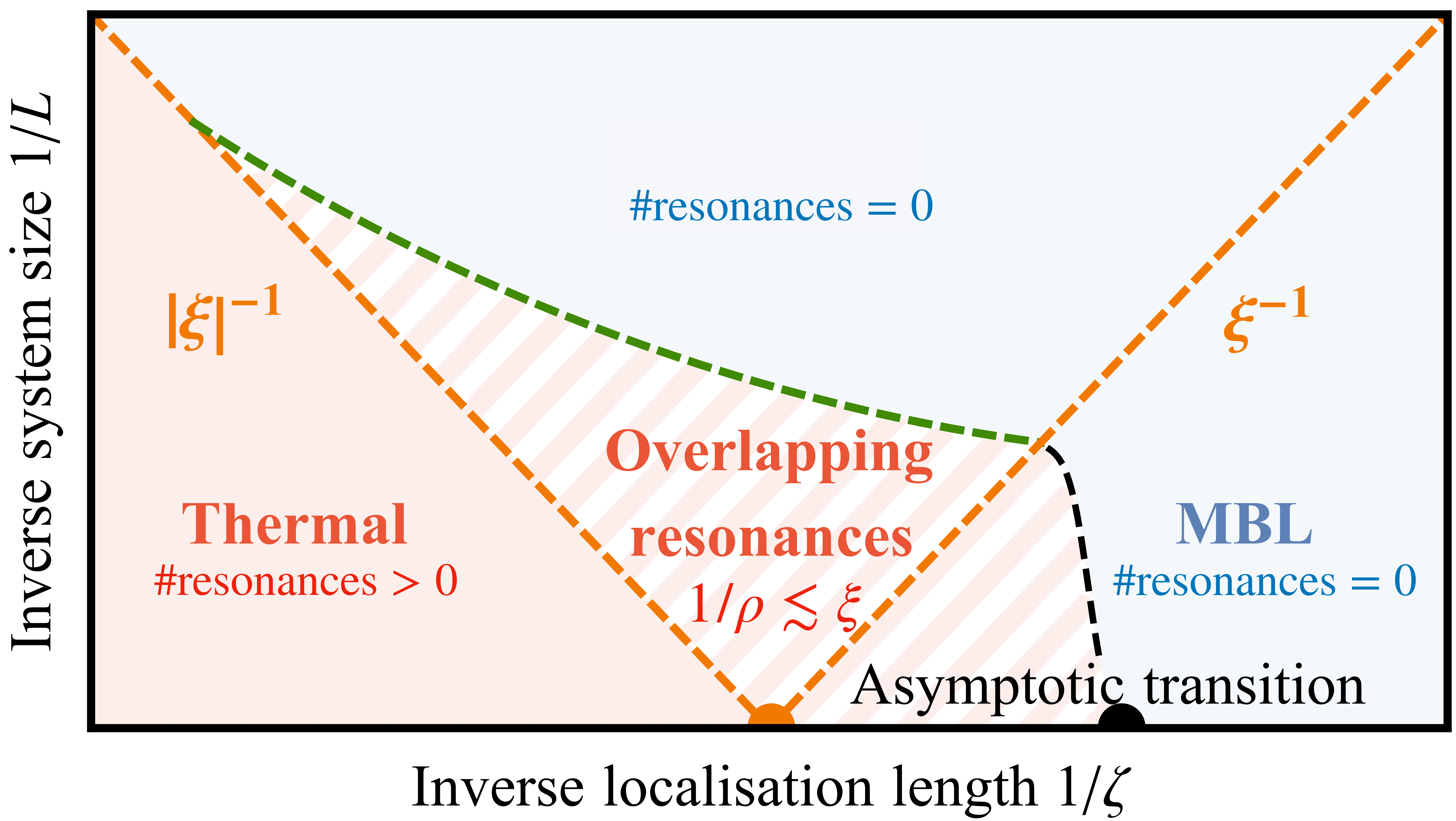
Resonance length

Pre-asymptotic scaling theory

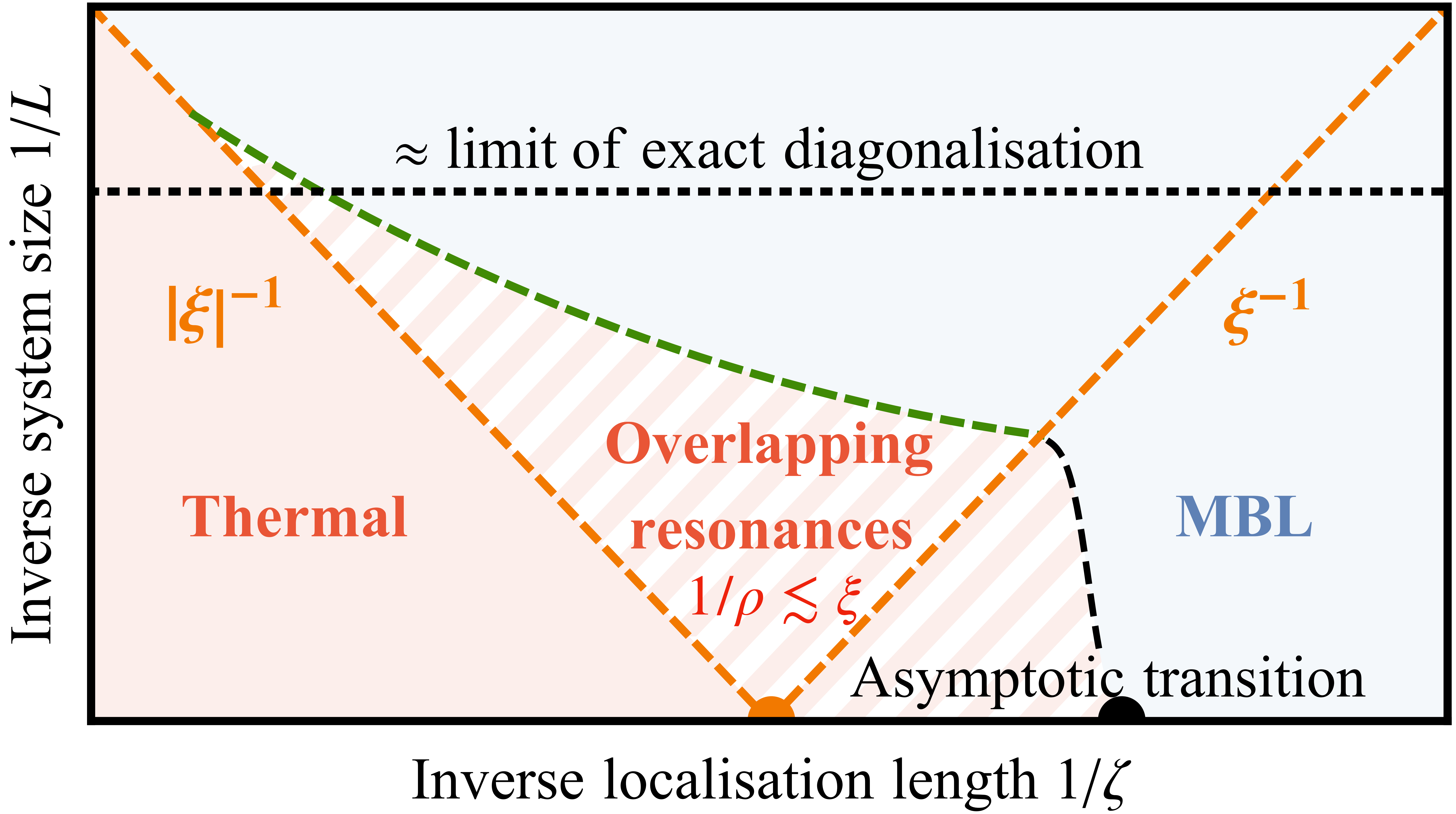
$q(r)$ = probability that a state a resonance at range r



Pre-asymptotic scaling theory



Pre-asymptotic scaling theory



The resonance model - implications

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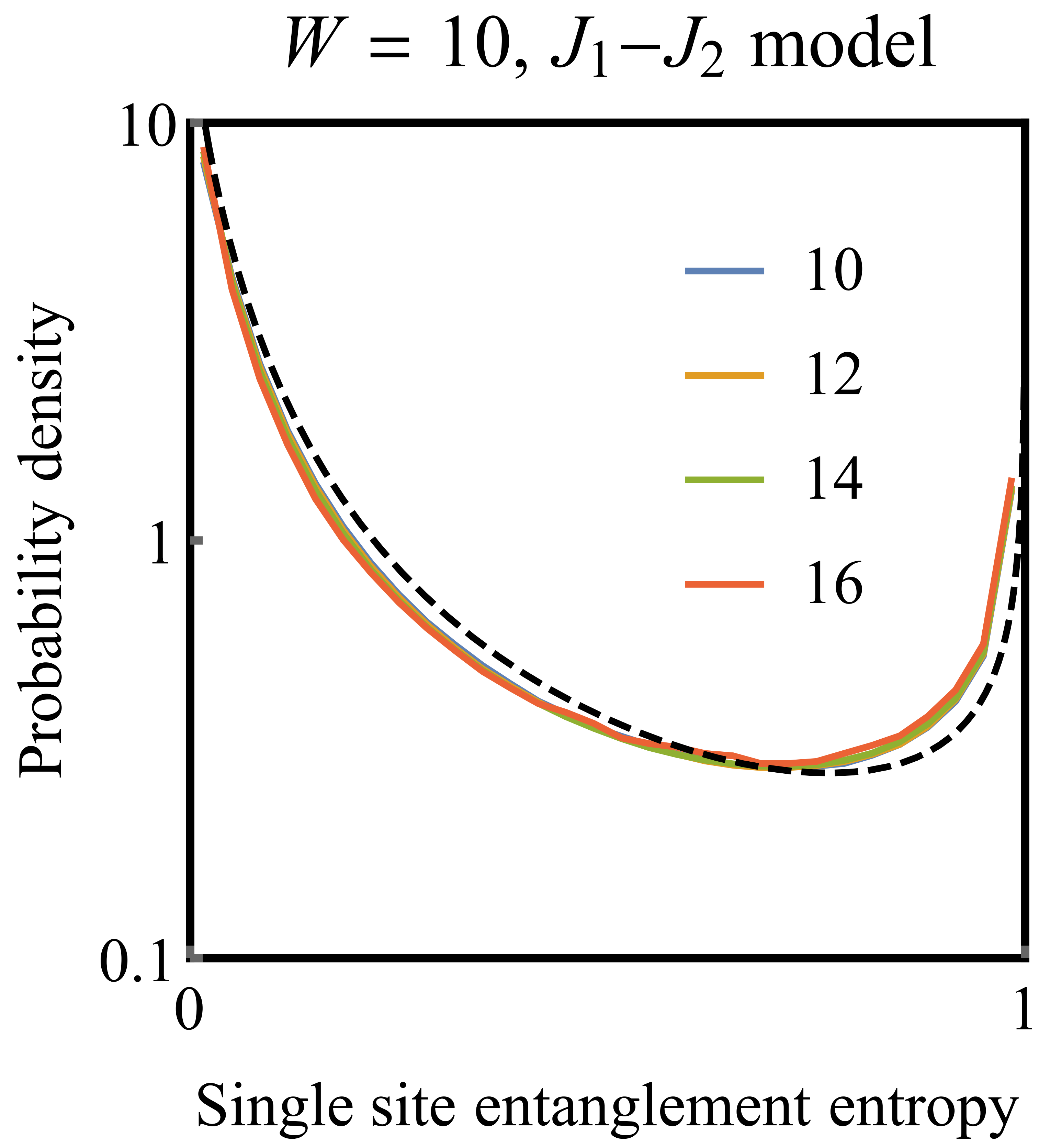
- "Sub-diffusion" without rare regions[1]
  - Quasiperiodic
  - Floquet
  - Thermal phase

$$|[H, \tilde{\tau}_z]| \sim \omega_\xi$$

- Exponentially diverging "Thouless time"[2]
- Spectral function $S(\omega) \sim \omega^{-1} \cdot O(\log^{1/2}(\omega))$
- Localised critical point[3]
- $\nu = 1$ scaling theory[4]
  - Random/Quasi-periodic/Hyper-uniform
- Scale free resonances[5]
- Maximal chaos[6]



1. Agarwal et al (2014), Gopalakrishnan et al (2015), Agarwal et al, Gopalakrishnan et al (2016), Bordia et al (2017), Schulz et al (2020), 2. Sels and Polkovnikov (2020), Lezama et al (2021), Tikhonov and Mirlin (2021), 3.Khemani et al (2017), 4. Kjäll et al (2014), Luitz et al (2015), 5. Villanlonga and Clark (2020), 6. Sels and Polkovnikov (2020)

Summary

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Assumptions

- MBL
- Small systems
- No rare regions

Resonance Model

- Model for eigenstates
- Instability of typical localised regions
- Pre-asymptotic scaling theory for the MBL-thermal crossover

Predictions

- Phenomenology of numerical crossover
- Self consistently stable MBL