Local resonances and parametric level dynamics in the many-body localised phase

arXiv:2107.12387

Sam Garratt
with Sthitadhi Roy and John Chalker

MBL2021: dead or alive?

Many-body localised phase

Theory based on existence of local integrals of motion (LIOM).

Serbyn, Papic, Abanin PRL 2013, Huse, Nandkishore, Oganesyan PRB 2014

Example: disordered Heisenberg chain

$$H = \sum_j \vec{h}_j \cdot \vec{\sigma}_j + J \sum_j \vec{\sigma}_j \cdot \vec{\sigma}_{j+1}$$

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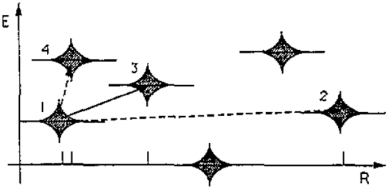
For small $J \neq 0$, LIOM $\tilde{\tau}_j^z = \tau_j^z +$ exponential tails

Resonances

In Anderson localised phase



Kramer & MacKinnon '93 Rep. Prog. Phys.

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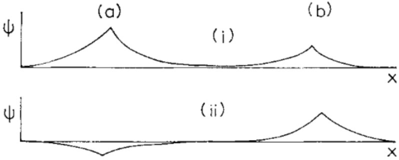

Mott '68 Philos. Mag.

Analogous phenomena in MBL phase

- Gopalakrishnan et al. PRB 92 104202 (2015)
- Imbrie PRL 117 027201 (2016)
- Crowley & Chandran arXiv:2012.14393
- Morningstar et al. arXiv:2107.05642

Overview

- Vary disorder realisation to induce resonances.

  Avoided level crossings = resonances

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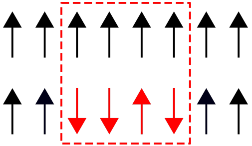

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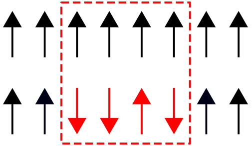

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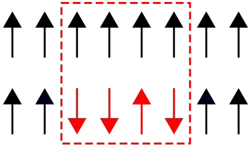

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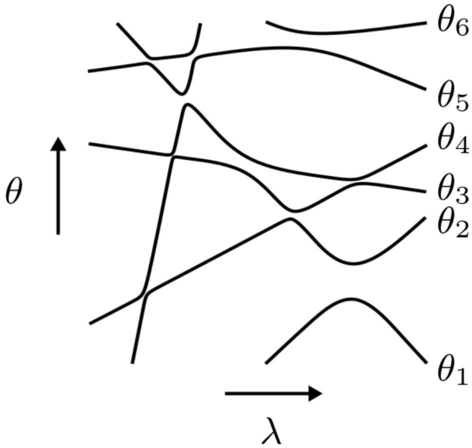
  Distributions of matrix elements of local operators.

- Capture behaviour not usually considered 'localised'.

Avoided crossings as resonances



From here: **Floquet** analogue of disordered Heisenberg chain

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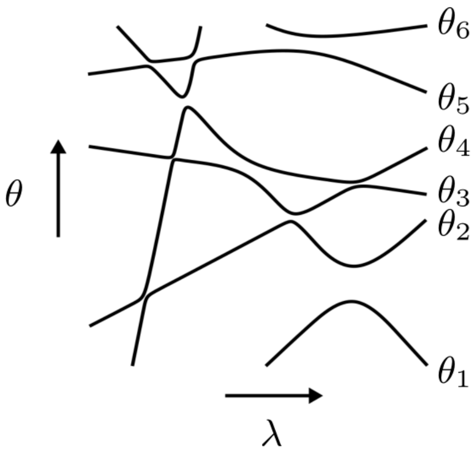

From here: **Floquet** analogue of disordered Heisenberg chain

Floquet operator W, with $W\,|n\rangle = e^{i\theta_n}\,|n\rangle$, $n = 1 \ldots 2^L$

$\theta_n =$ quasienergy, average d.o.s. $\sum_n \langle \delta_{2\pi}(\theta - \theta_n) \rangle = [2\pi]^{-1} \times 2^L$.

Level curvatures

Distribution of level curvatures $\kappa_n = \partial_\lambda^2 \theta_n$. At large κ

$$p_\kappa(\kappa) \sim L|\kappa|^{-(2-\zeta \ln 2)}$$

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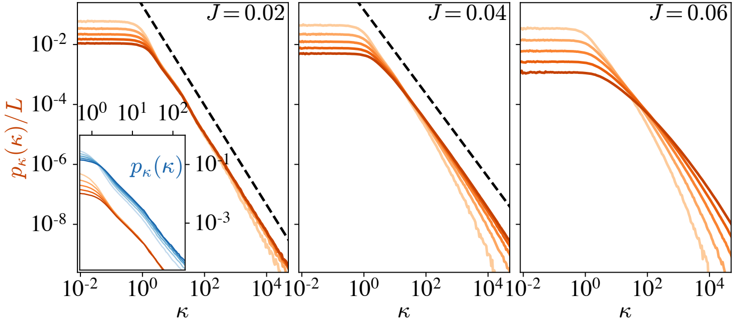

Local observables

LIOM at $J = 0$ are single-site $\tau_j^z = (\vec{h}_j / h_j) \cdot \vec{\sigma}_j$.
$\implies$ For $J = 0$, $\langle n | \tau_j^z | m \rangle = 0$ for $n \neq m$.

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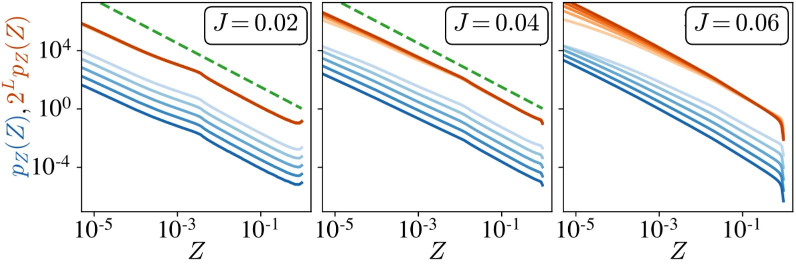

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- From theory + numerics on physical quantities, infer ζ

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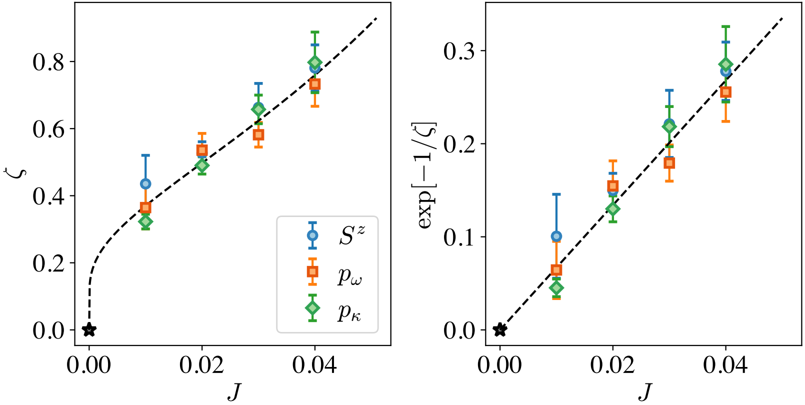

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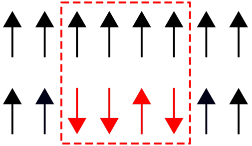

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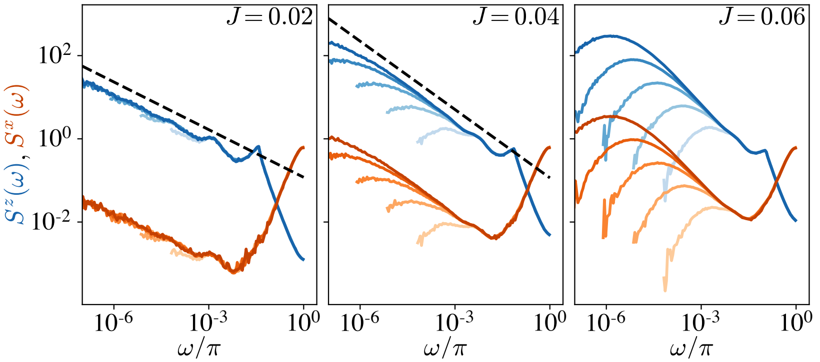
Spectral function see also Gopalakrishnan et al. PRB 2015, Crowley & Chandran arXiv:2012.14393

$$\langle S^z(\omega) \rangle \sim \omega^{-\zeta \ln 2}$$



Spectral statistics

Two-point correlator of the level density

$$p_\omega(\omega) = [2\pi]^{-1}\left[1 - a\frac{L}{2^L}\omega^{-\zeta \ln 2} + \dots\right]$$