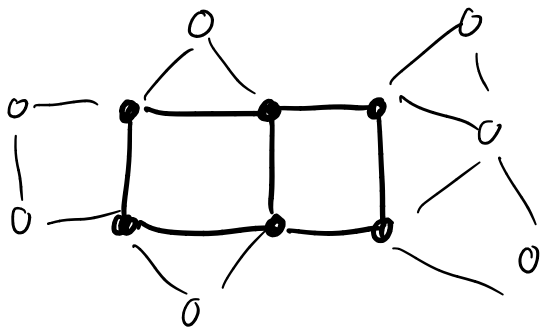
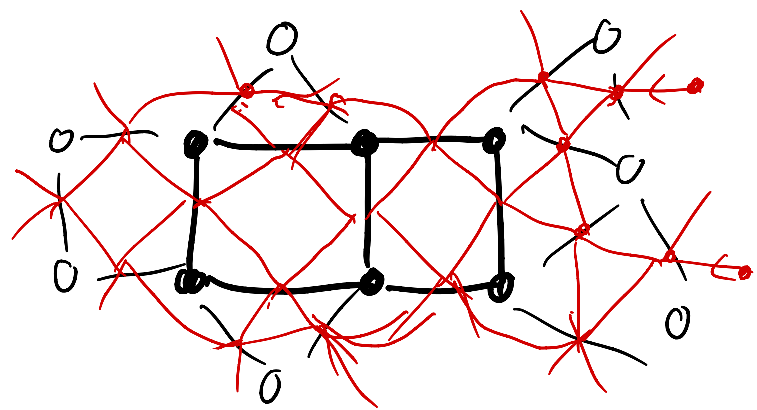
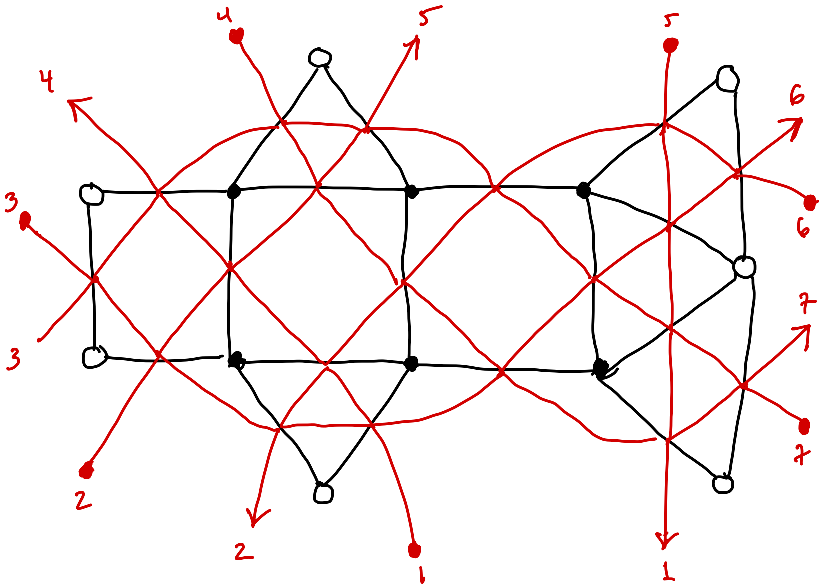
Today) Running in $Gr(3,7)$

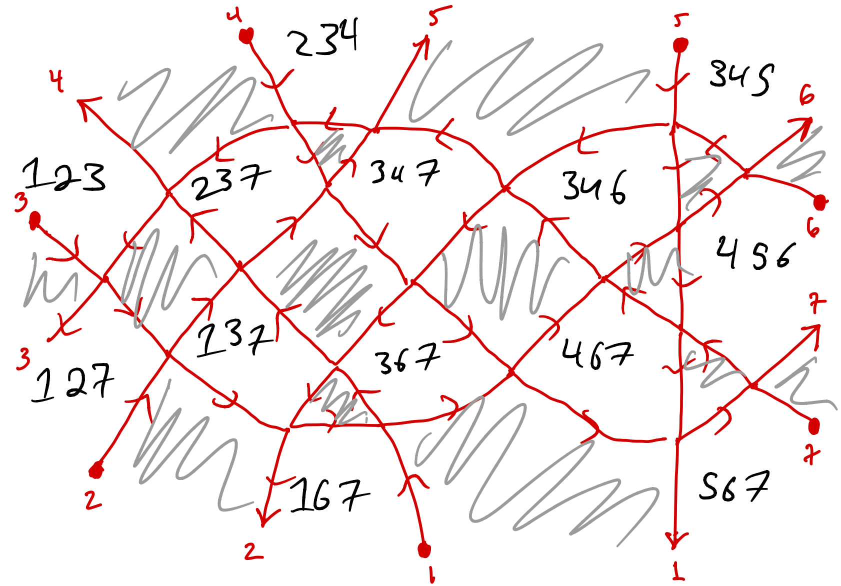
Take $(3-1) \times (7-3-1)$ grid graph & add $\overset{7}{\sim}$ boundary nodes, so that all vx have even valency & internal have val ≥ 4

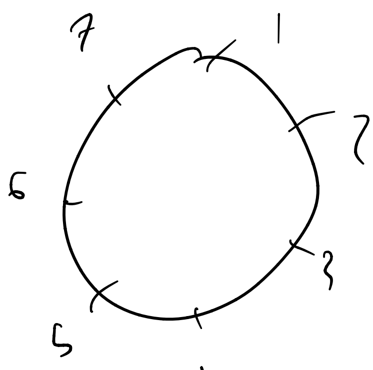


Dualize









$\pi_{3,7}$

Label an alt region w/ i if it lies to left of strand from i

Twists

$$M = (v_1 \quad v_2 \quad v_3 \quad v_4 \quad v_5 \quad v_6 \quad v_7) \in Gr(3,7)$$

$$\overleftarrow{M} = (v_6 \times v_7,\ v_7 \times v_1,\ v_1 \times v_2,\ v_2 \times v_3,\ v_3 \times v_4,\ v_4 \times v_5,\ v_5 \times v_6)$$

$$[\overleftarrow{347}] = \det(v_1 \times v_2,\ v_2 \times v_3,\ v_5 \times v_6)$$

$$= \langle v_1 \times v_2,\ (v_2 \times v_3) \times (v_5 \times v_6) \rangle$$

$$= \det \begin{pmatrix} \langle v_1, v_2 \times v_3 \rangle & \langle v_1, v_5 \times v_6 \rangle \\ \underset{0}{\cancel{\langle v_2, v_2 \times v_3 \rangle}} & \langle v_2, v_5 \times v_6 \rangle \end{pmatrix} = [123][256]$$

Alternative Computation

$\sigma(i) \equiv i-1 \mod n$

$\underline{i} = \{\sigma^{k-1}(i), \ldots, \sigma(i), i\}$

Prop 3.5 [Marsh-Scott] $I = I_1 \cup I_2$

$$I_1 = \{\sigma^p(i), \ldots, \sigma(i), i\}$$

$p+q+2 = k$

$p, q \geqslant 0$

$$I_2 = \{\sigma^q(j), \ldots, \sigma(j), j\}$$

$$[\overleftarrow{I}] = [J] \prod_{r=1}^{p} \sigma_r(\underline{i}) \prod_{r=1}^{q} \sigma_r(\underline{j})$$

cyclic intervals length k ending all elts of I except i, j

$$J = J_1 \cup J_2$$

$$J_1 = \{\sigma^{p+q+1}(i), \ldots, \sigma^{p+1}(i)\}$$

$$J_2 = \{\sigma^{p+q+1}(j), \ldots, \sigma^{q+1}(j)\}$$

Prop 3.5 [Marsh- Scott] $I = I_1 \cup I_2$

$I_1 = \{ \sigma^{p}(i), \dots \sigma(i), i \}$

$p+q+2 = k$
$p, q \geq 0$

$I_2 = \{ \sigma^{q}(j), \dots, \sigma(j), j \}$

$$\overleftarrow{[I]} = [J] \underbrace{\prod_{r=1}^{p} [\sigma_r(\underline{i})] \prod_{r=1}^{q} [\sigma_r(\underline{j})]}$$

cyclic intervals length k ending all elts of I except i, j

$J = J_1 \cup J_2$ $J_1 = \{ \sigma^{p+q+1}(i), \dots \sigma^{p+1}(i) \}$

$J_2 = \{ \sigma^{p+q+1}(j), \dots, \sigma^{q+1}(j) \}$

Check w/ [3 4 7]

$\{3, 4, 7\} = \{3, 4\} \sqcup \{7\}$

$i = 4$, $p = 1$ $j = 7$, $q = 0$

$J_1 = \{2\}$

$J_2 = \{5, 6\}$

$$\overleftarrow{[3 4 7]} = [2 5 6][1 2 3]$$

[346] → [245] [123]

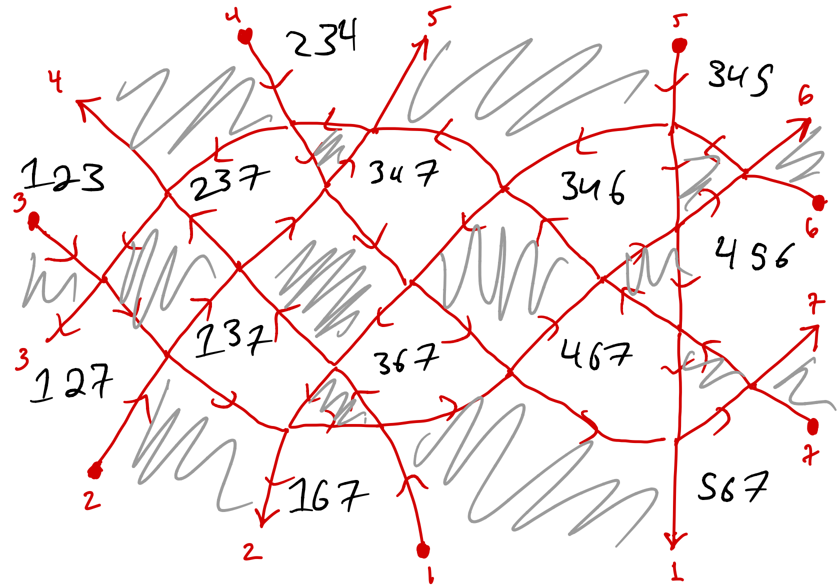
[467] → [235] [456]

[367] → [125] [456]

[237] → [156] [123]

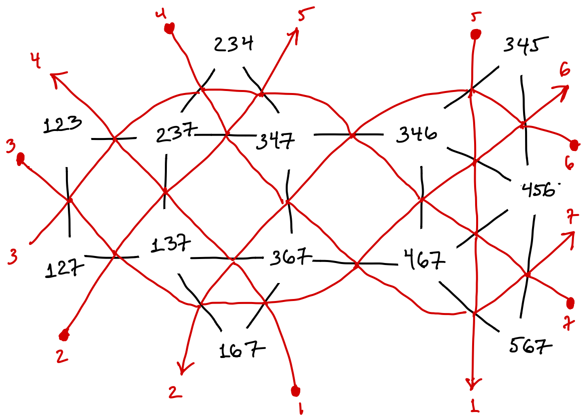
[347] → [256] [123]

[137] → [126] [567]



2× 34	2× 56
2× 67	2× 12
1× 71	1× 23
1× 23	1× 45

D


234
123
237
347
346
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456
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167

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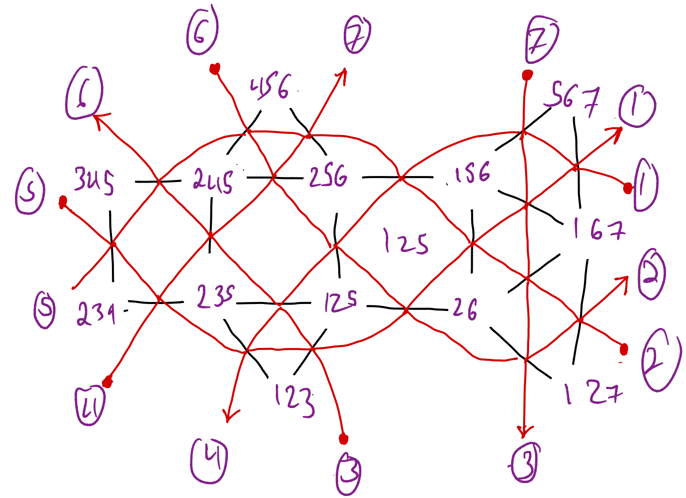
456
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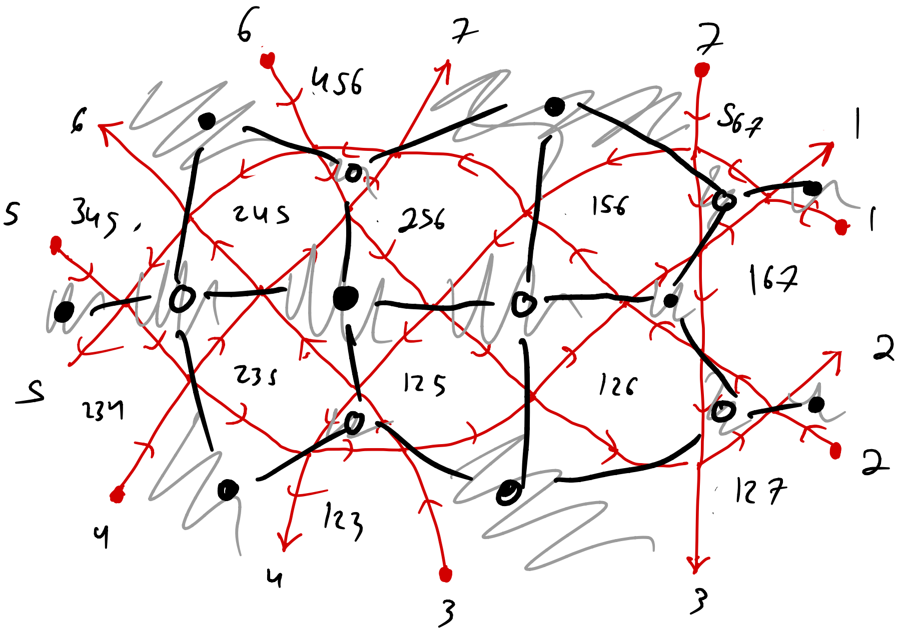
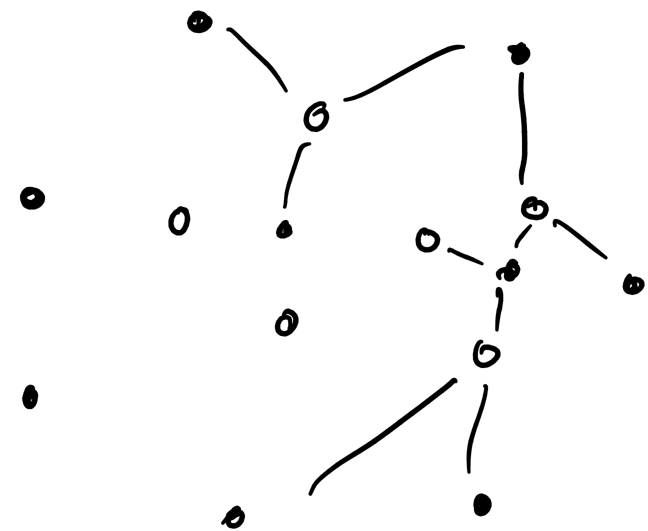
Recall dual bipartite graph

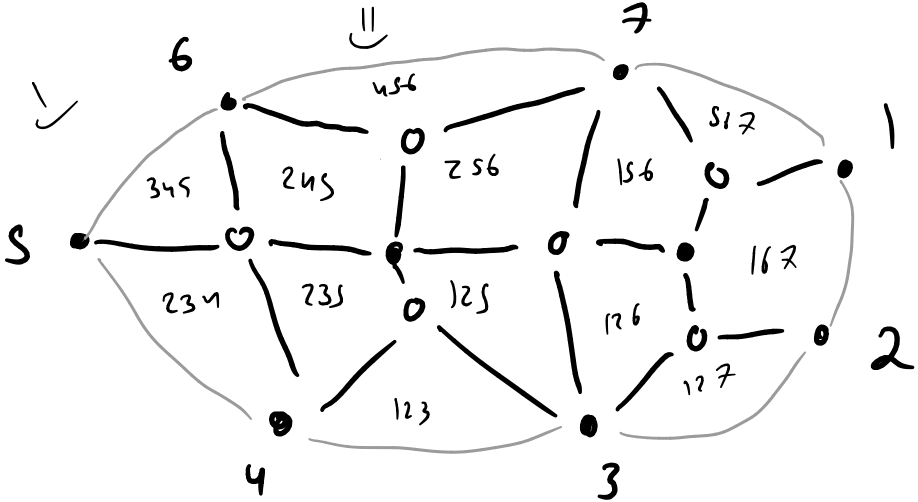
● – CW

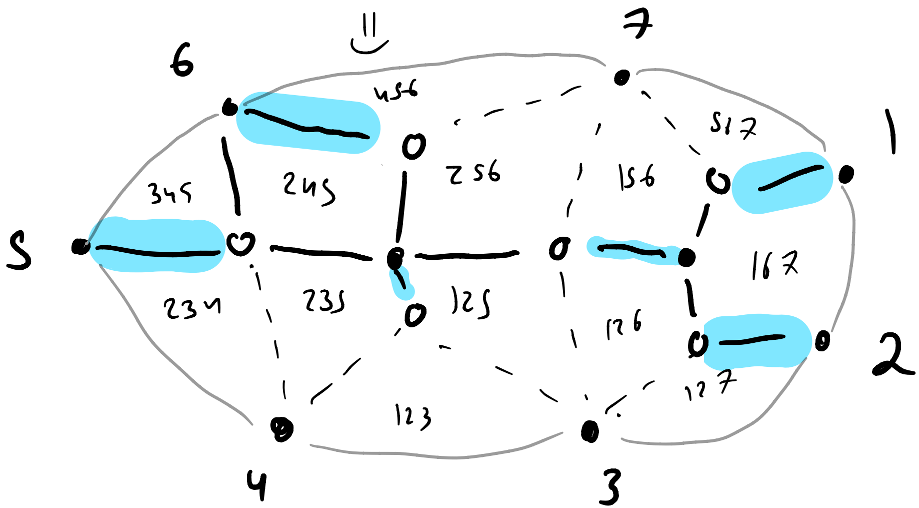
o – CCW



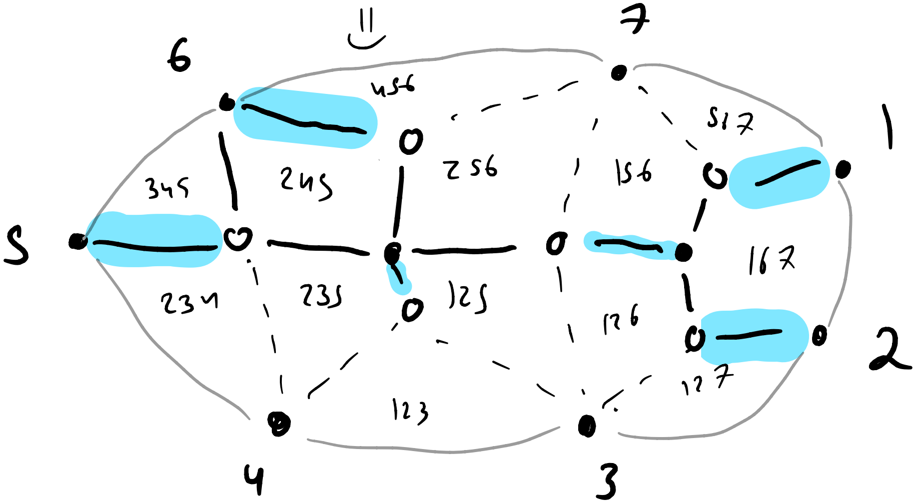


$\Gamma_{\check{D}}$ ✓

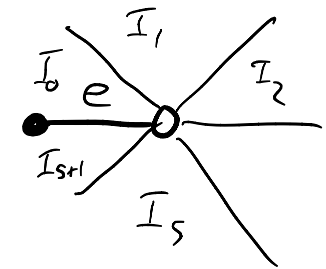




Claim) There is exactly one dimer cover of $\Gamma_{\check{D}}^{I}$ (remove ∂ vertices w/ labels from I) for I from D



Recall)



$$\mathrm{wt}(e) = [I_1][I_2]\cdots[I_s]$$

$$\text{wt of dimer}) = [245][235][256]^2[156][126][125]\quad[123]$$

$$\Delta^{[347]}_{\check{D}} = \prod_{\substack{f \text{ internal} \\ \text{face} \\ \text{of } \check{D}}} [f] \cdot [256][123] = \prod_{\check{D}} [347]$$