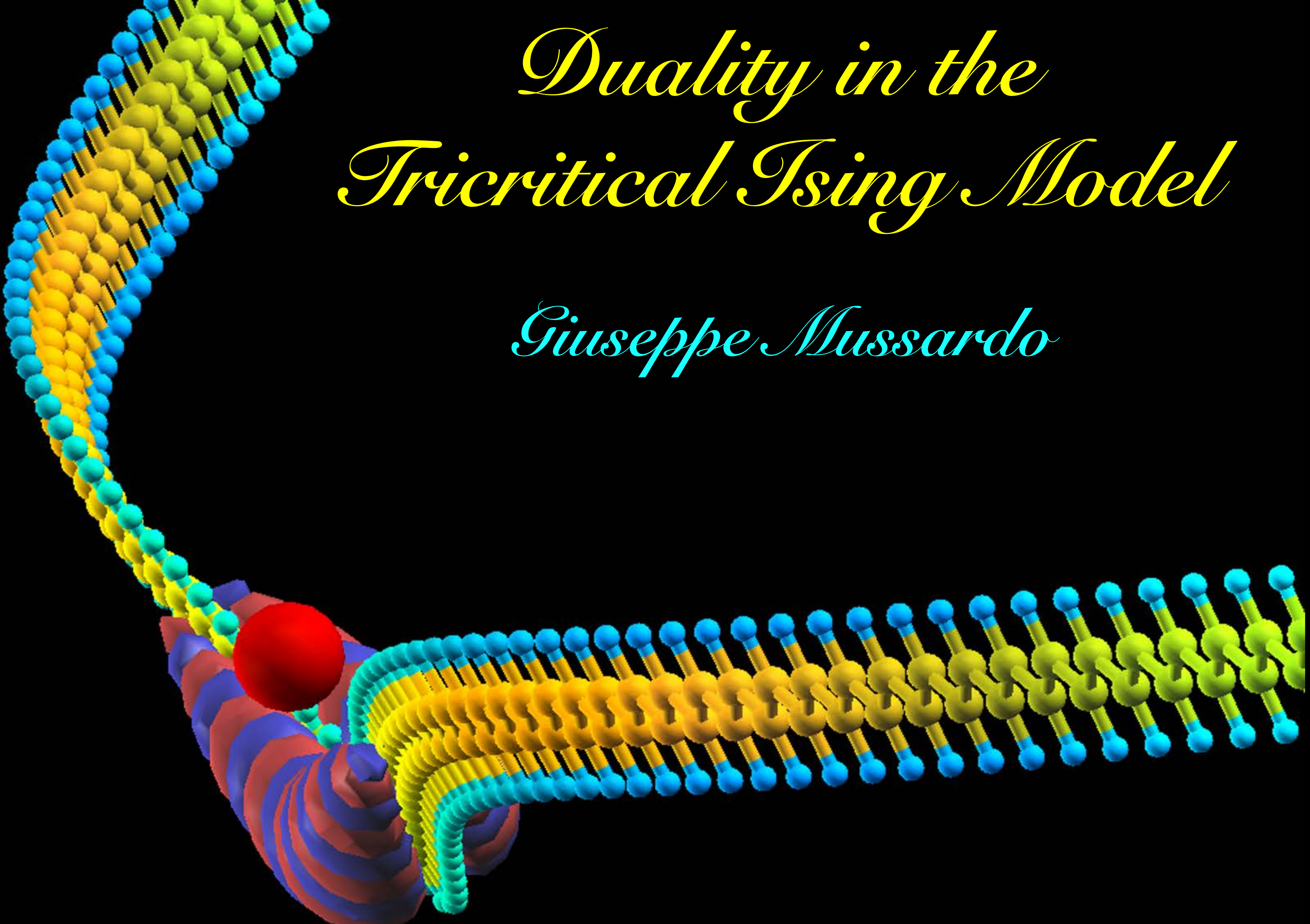
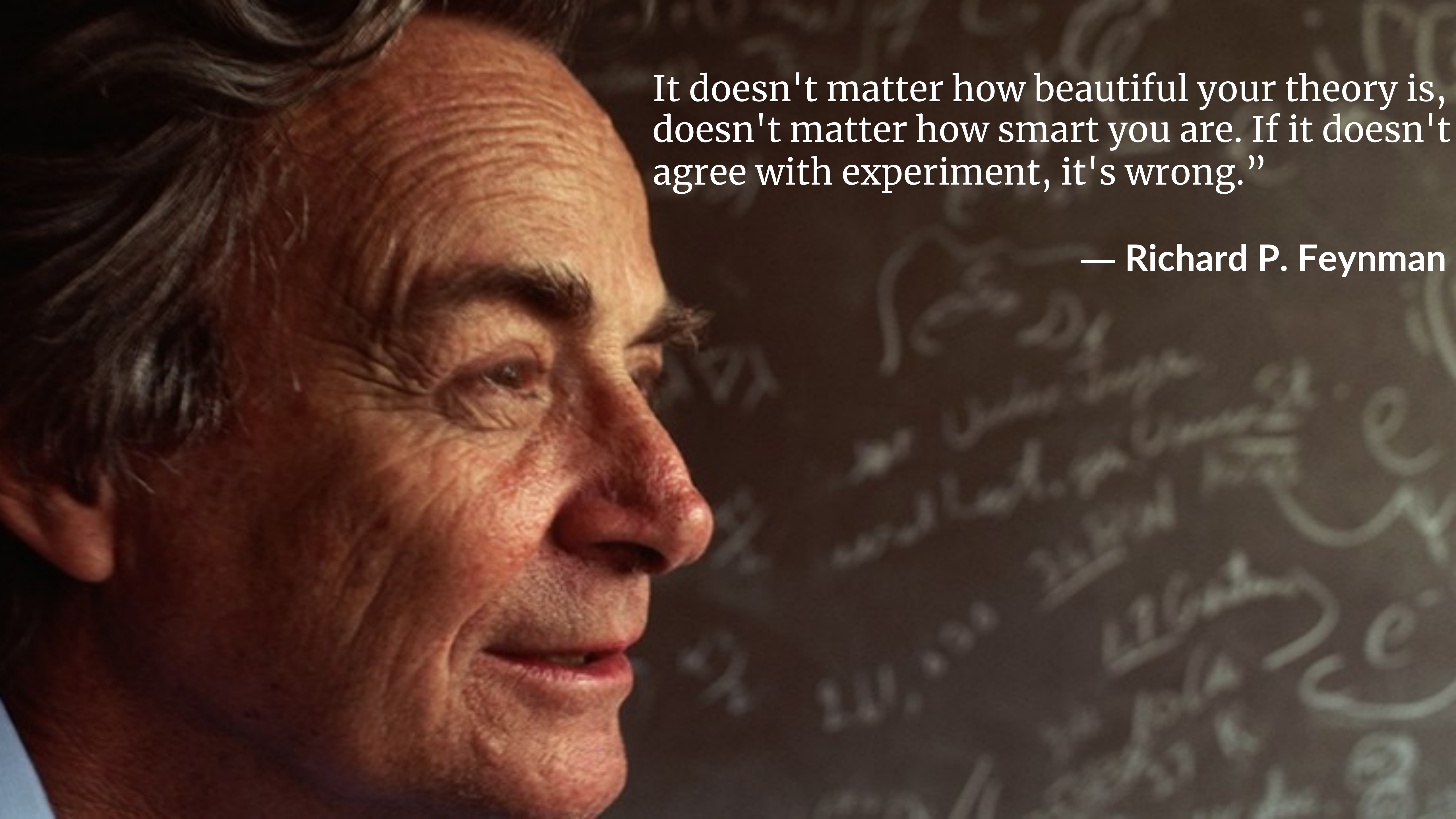
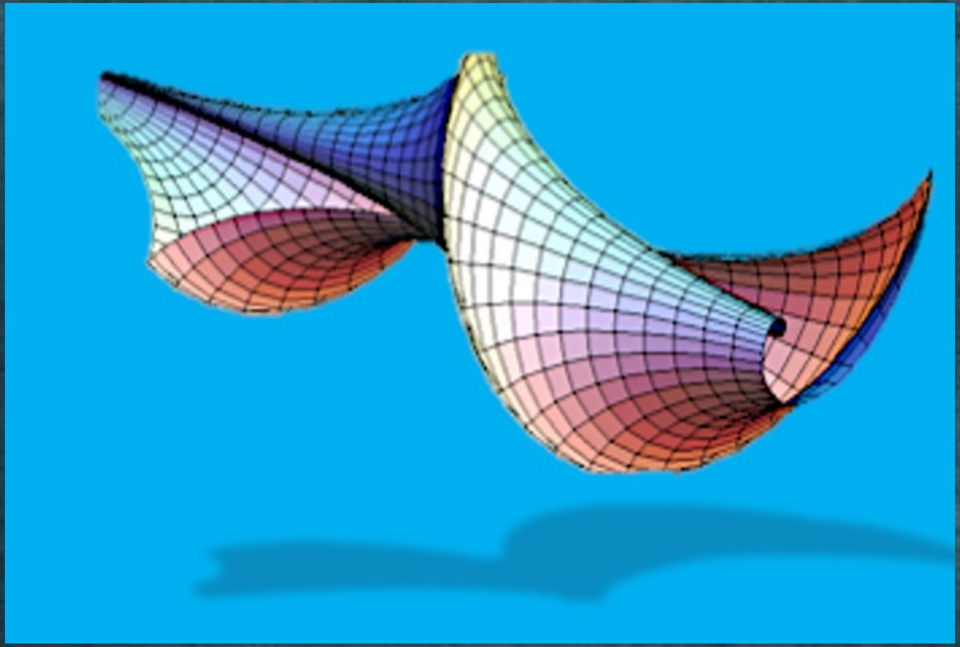
Duality in the Tricritical Ising Model

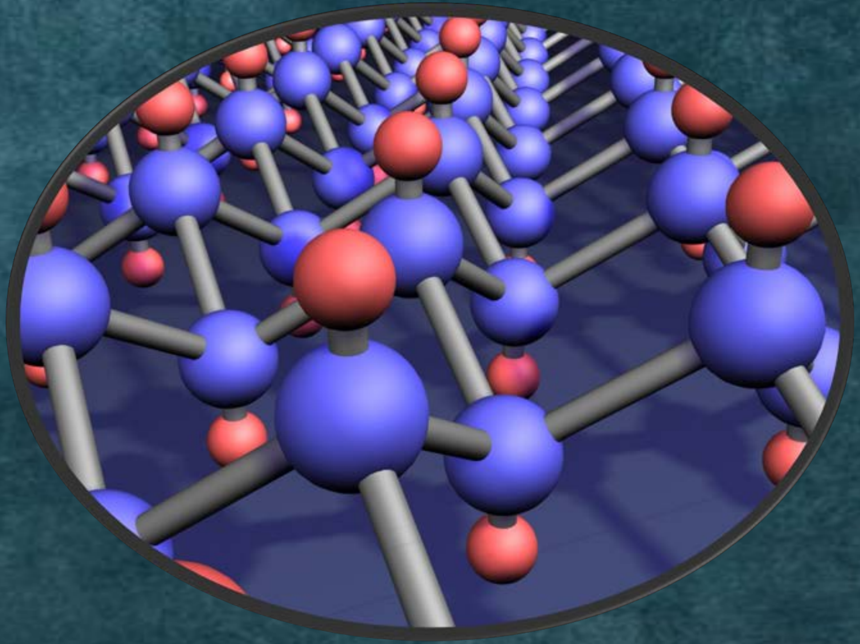
Giuseppe Mussardo



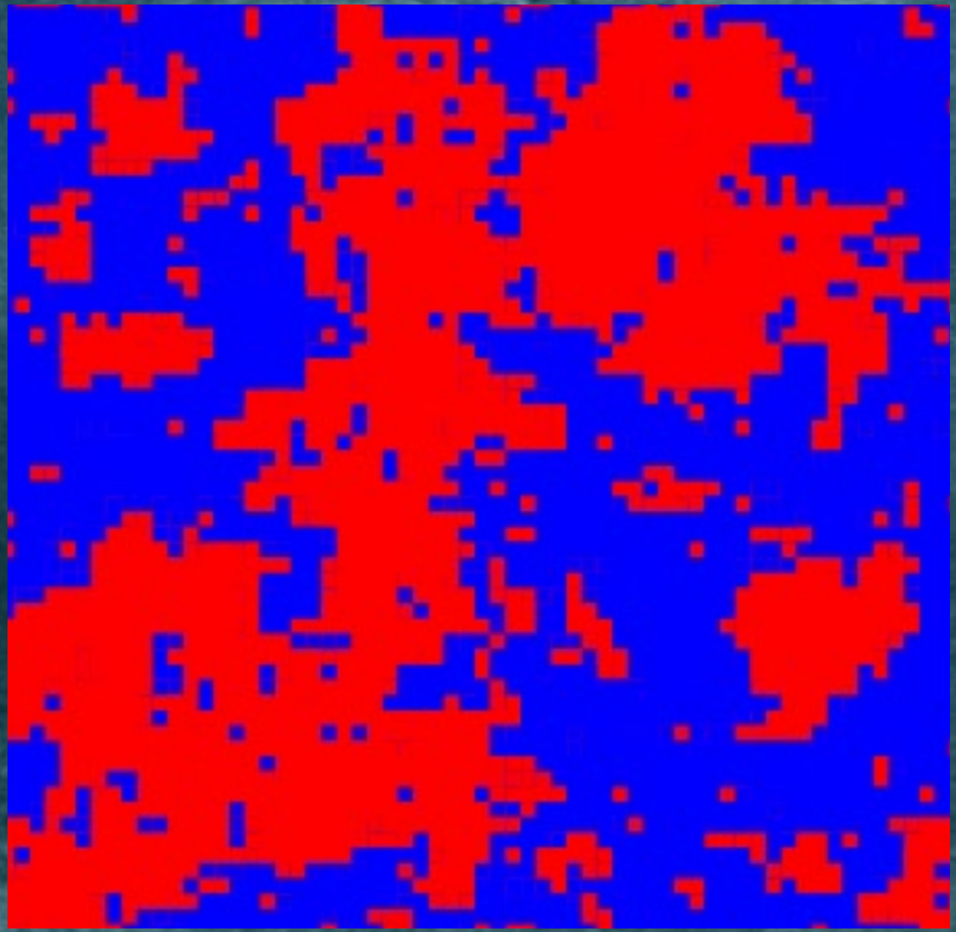

It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong.”
— Richard P. Feynman

In recent years, there has been a stunning and a delightful matching between the beautiful theoretical and experimental worlds of quantum integrable models and experimental quantum chains !!

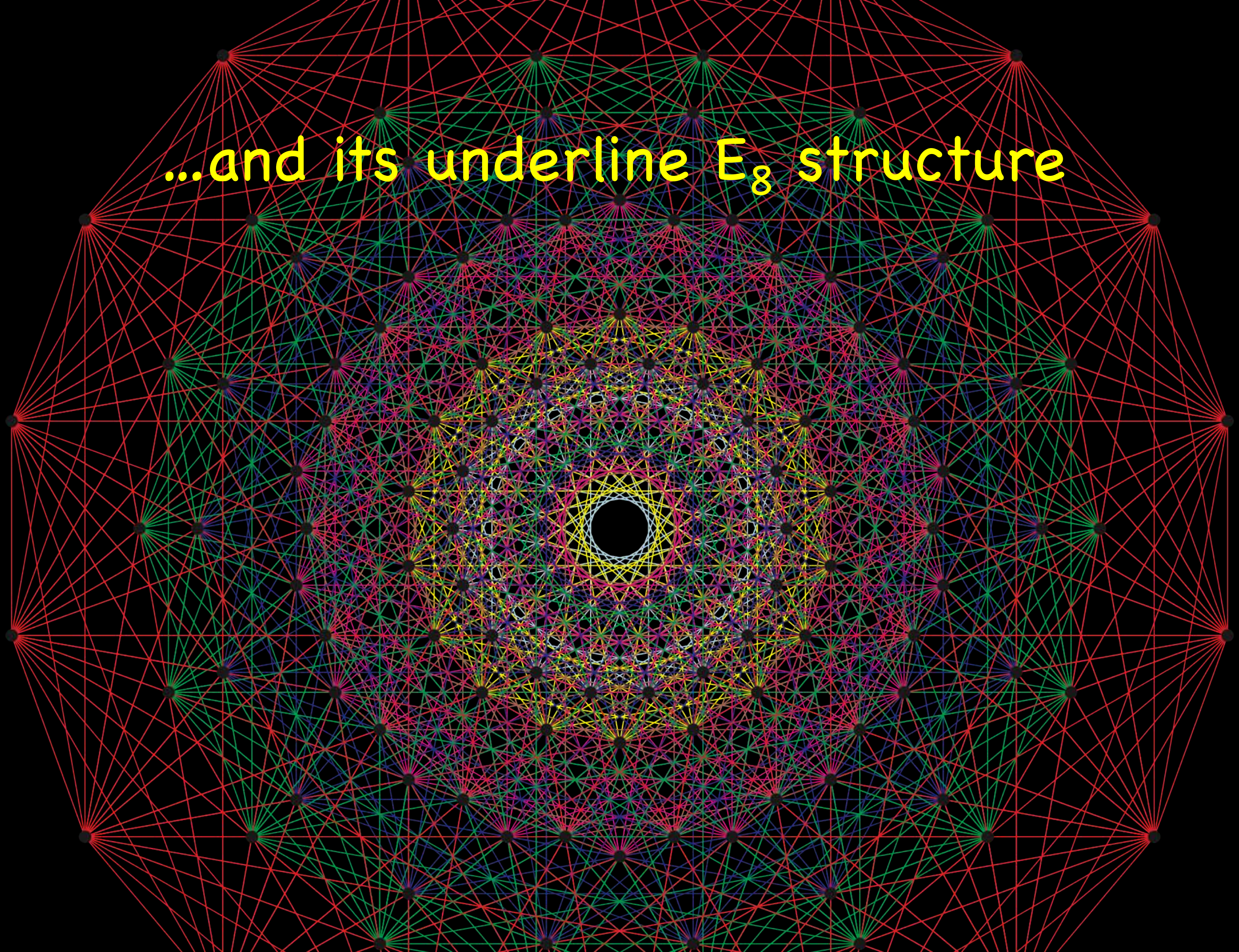




A remarkable example: 2d Ising model in a magnetic field at $T=T_c$ ...



…and its underline E_8 structure



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INTEGRALS OF MOTION AND S-MATRIX OF THE (SCALED) $T = T_c$ ISING MODEL WITH MAGNETIC FIELD

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It is shown that the field theory describing the scaling limit of $T = T_c$ Ising model with nonzero magnetic field possesses a number of nontrivial local integrals of motion. The exact mass spectrum and S-matrix of this field theory is conjectured.

1. Introduction

Conformal field theory (CFT) and Integrable field theory (IFT) in two dimensions are two subjects which attracted much attention in the last years. The subjects seem to be deeply related. Very close mathematics is involved in treatment of both theories (see Ref. 1 and references therein). Also, the ultraviolet limit (and sometimes also the infrared limit[2,3]) of IFT is described by CFT and so the general IFT can be considered as the CFT perturbed by particular "integrable" relevant operator.[4,5] The most simple example is the scaling limit $T \to T_c$ of the Ising model (with zero magnetic field), which is well known to be the (certainly integrable) field theory of free massive Majorana fermions; it can be considered as $c = 1/2$ CFT perturbed by the spinless primary field $\varepsilon = \Phi_{(1,3)}$ ("energy density") having the conformal dimensions $(1/2, 1/2)$.[a]

In this paper we consider the same $c = 1/2$ CFT but now perturbed by the Z_2 odd primary operator $\sigma = \Phi_{(1,2)}$

$$H_{1/2}^{(1,2)} = H_{1/2} + h \int \sigma(x) d^2x \tag{1.1}$$

where $H_{1/2}$ is the Action (or Hamiltonian in statistical physics) of the $c = 1/2$ CFT and h is the (dimensional) constant. The field $\sigma(x)$ (which have the dimensions $(1/16, 1/16)$) is interpreted as the spin density in the critical $T = T_c$ Ising model and so the Hamiltonian (1.1) describes the scaling limit of $T = T_c$ Ising model with nonzero magnetic field h. It will be shown that the field theory (1.1) possesses several nontrivial local integrals of motion (IM) of the form

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[a] The algebraic structure of this off-critical IFT and its connection with the algebraic structure of CFT is

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The spin–spin correlation function in the two-dimensional Ising model in a magnetic field at $T = T_c$

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Abstract

The form-factor bootstrap approach is used to compute the exact contributions in the large-distance expansion of the correlation function $\langle\sigma(x)\sigma(0)\rangle$ of the two-dimensional Ising model in a magnetic field at $T = T_c$. The matrix elements of the magnetization operator $\sigma(x)$ present a rich analytic structure induced by the (multi-) scattering processes of the eight massive particles of the model. The spectral representation series has a fast rate of convergence and perfectly agrees with the numerical determination of the correlation function.

1. Introduction

Over the past few years, considerable progress has been made in the use of conformal invariance methods and scattering theory for the understanding of the critical points and the nearby scaling region of two-dimensional statistical models (see, for instance Refs. [1,2]). At the critical points, the correlation functions of the statistical models fall into a scale-invariant regime and their computation may be achieved by solving the linear differential equations obtained by the representation theory of the infinite-dimensional conformal symmetry [3]. The situation is different away from criticality. The scaling region may be described in terms of the relevant deformations of the fixed point actions. These deformations destroy the long-range fluctuations of the critical point and the associated quantum field theories are usually massive. If an infinite number of conserved charges survive the deformation of the critical point action, the corresponding QFT can be efficiently characterized (on-shell) by the relativistic scattering processes of

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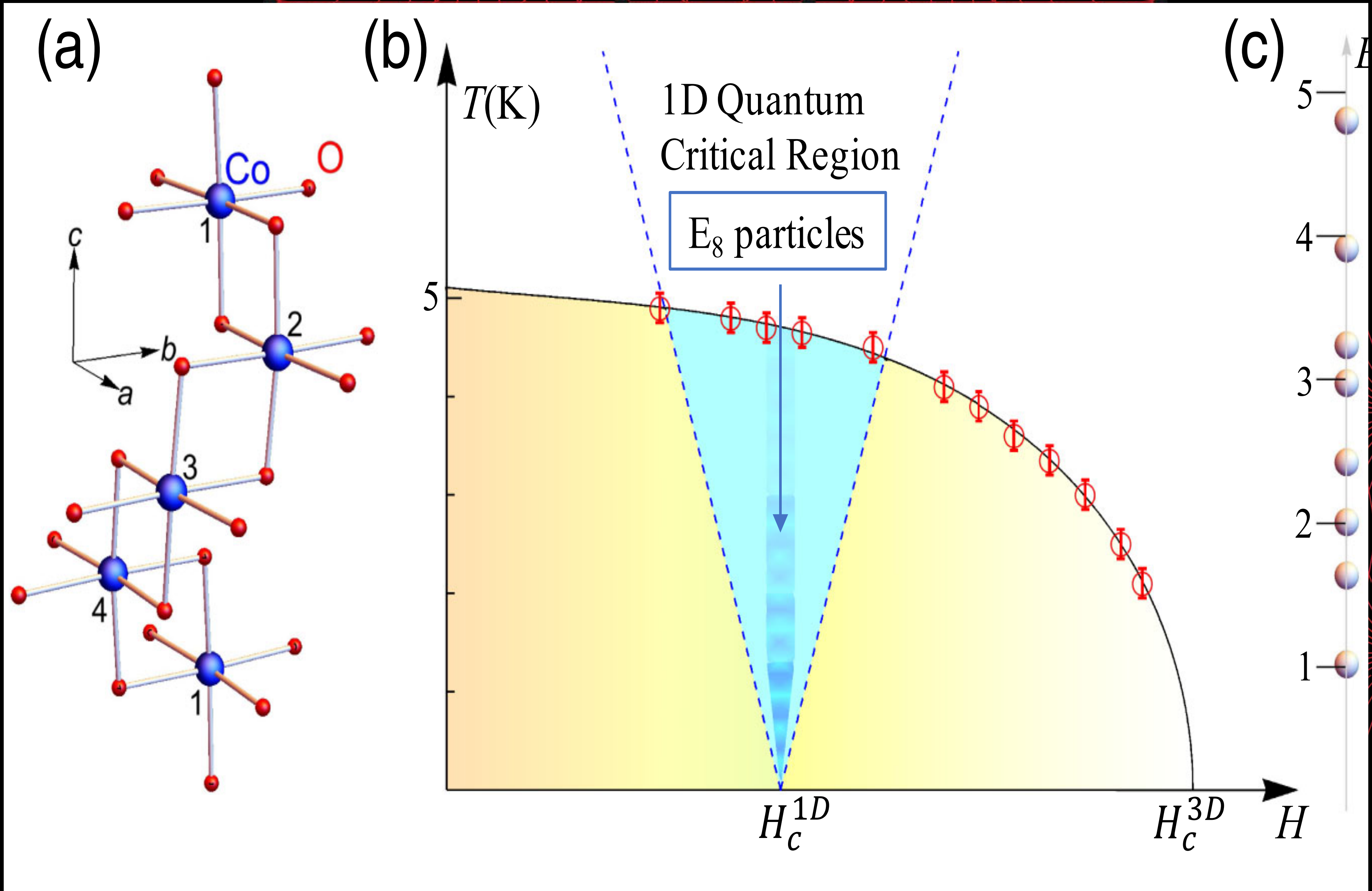

Spin-chain material $BaCo_2V_2O_8$

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Supporting Online Material

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Materials and Methods
Figs. S1 to S18
Table S1
References

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Quantum Criticality in an Ising Chain: Experimental Evidence for Emergent E_8 Symmetry

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Quantum phase transitions take place between distinct phases of matter at zero temperature. Near the transition point, exotic quantum symmetries can emerge that govern the excitation spectrum of the system. A symmetry described by the E_8 Lie group with a spectrum of eight particles was long predicted to appear near the critical point of an Ising chain. We realize this system experimentally by using strong transverse magnetic fields to tune the quasi–one-dimensional Ising ferromagnet $CoNb_2O_6$ (cobalt niobate) through its critical point. Spin excitations are observed to change character from pairs of kinks in the ordered phase to spin-flips in the paramagnetic phase. Just below the critical field, the spin dynamics shows a fine structure with two sharp modes at low energies, in a ratio that approaches the golden mean predicted for the first two meson particles of the E_8 spectrum. Our results demonstrate the power of symmetry to describe complex quantum behaviors.

Symmetry is present in many physical systems and helps uncover some of their fundamental properties. Continuous symmetries lead to conservation laws; for example, the invariance of physical laws under spatial rotation ensures the conservation of angular momentum. More exotic continuous symmetries have been predicted to emerge in the proximity of certain quantum phase transitions (QPTs) (*1*, *2*). Recent experiments on quantum magnets (*3*–*5*) suggest that quantum critical resonances may expose the underlying symmetries most clearly. Remarkably, the simplest of systems, the Ising chain, promises a very complex symmetry, described mathematically by the E_8 Lie group (*2*, *6*–*9*). Lie groups describe continuous symmetries and are important in many areas of physics. They range in complexity from the U(1) group, which appears in the low-energy description of superfluidity, superconductivity, and Bose-Einstein condensation (*10*, *11*), to E_8, the highest-order symmetry group discovered in mathematics (*12*), which has not yet been experimentally realized in physics.

The one-dimensional (1D) Ising chain in transverse field (*10*, *11*, *13*) is perhaps the most-studied theoretical paradigm for a quantum phase transition. It is described by the Hamiltonian

$$H = \Sigma_i - JS_i^z S_{i+1}^z - hS_i^x \quad (1)$$

where a ferromagnetic exchange $J > 0$ between nearest-neighbor spin-½ magnetic moments $\mathbf{S}_i$ arranged on a 1D chain competes with an applied external transverse magnetic field h. The Ising exchange J favors spontaneous magnetic order along the z axis ($|\uparrow\uparrow\uparrow \cdots \uparrow\rangle$ or $|\downarrow\downarrow\downarrow \cdots \downarrow\rangle$), whereas the transverse field h forces the spins to point along the perpendicular $+x$ direction ($|\rightarrow\rightarrow\rightarrow \cdots \rightarrow\rangle$). This competition leads to two distinct phases, magnetically ordered and quantum paramagnetic, separated by a continuous transition at the critical field $h_C = J/2$ (Fig. 1A). Qualitatively, the magnetic field stimulates quantum tunneling processes between ↑ and ↓ spin states and these zero-point quantum fluctuations "melt" the magnetic order at h_C (*10*).

To explore the physics of Ising quantum criticality in real materials, several key ingredients are required: very good one-dimensionality of the magnetism to avoid mean-field effects of higher dimensions, a strong easy-axis (Ising) character, and a sufficiently low exchange energy J of a few meV that can be matched by experimentally attainable magnetic fields (10 T ~ 1 meV) to access the quantum critical point. An excellent model system to test this physics is the insulating quasi-1D Ising ferromagnet $CoNb_2O_6$ (*14*–*16*), where magnetic Co^{2+} ions are arranged into near-isolated zigzag chains along the c axis with strong easy-

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Rapid Communications

Observation of E_8 particles in an Ising chain antiferromagnet

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Near the transverse-field-induced quantum critical point of the Ising chain, an exotic dynamic spectrum consisting of exactly eight particles was predicted, which is uniquely described by an emergent quantum integrable field theory with the symmetry of the E_8 Lie algebra, but rarely explored experimentally. Here we use high-resolution terahertz spectroscopy to resolve quantum spin dynamics of the quasi-one-dimensional Ising antiferromagnet $BaCo_2V_2O_8$ in an applied transverse field. By comparing to an analytical calculation of the dynamical spin correlations, we identify E_8 particles as well as their two-particle excitations.

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Exotic states of matter, such as high-temperature superconductivity or magnonic Bose-Einstein condensation, can emerge in the vicinity of a quantum critical point [1], which identifies a zero-temperature phase transition tuned by an external parameter, e.g., chemical substitution or applied magnetic field [2,3]. Quantum critical points are often characterized by enhanced many-body fluctuations together with divergence of correlation length and complex emergent symmetry [1,4–8]; thus it is generally a formidable task to precisely describe the quantum many-body physics near a quantum critical point. Exactly solvable models play a crucial role in this regard, because a precise understanding of the quantum many-body physics can be gained by rigorously analyzing these models [4,6]. The one-dimensional (1D) spin-1/2 Ising model in a transverse magnetic field is such a paradigmatic example [1,4–9]. Considering only the exchange interaction between the nearest-neighbor spins on a chain [10,11], this model has been investigated most broadly in quantum magnetism, which provides deep insights into the fundamental aspects of the quantum many-body physics [1,6–8]. In particular, highly unconventional dynamic properties have been theoretically predicted to emerge near the transverse-field Ising quantum critical point, either for equilibrium states upon constant perturbations or for states far from equilibrium after a quantum quench (see, e.g., Refs. [12–18]). Moreover, the study of the transverse-field Ising quantum critical point is of importance also in the context of quantum information [5,8] and quantum simulation using ultracold atoms [19].

A remarkable prediction of an exotic dynamic spectrum was made three decades ago for the transverse-field Ising chain perturbed by a small longitudinal field [12]. It is described by the Hamiltonian

$$H = -J \sum_i S_i^z S_{i+1}^z - B_\perp \sum_i S_i^x - B_\parallel \sum_i S_i^z, \qquad (1)$$

with the x and z components S_i^x and S_i^z, respectively, of the spin-1/2 magnetic moment at the ith site on a 1D chain. The first term is the Ising term with the ferromagnetic exchange $J > 0$ between the nearest-neighbor spins. The second and third terms describe the interactions of the spins with the transverse field $B_\perp$ and the perturbative longitudinal field $B_\parallel$, respectively. Close to the transverse-field Ising quantum critical point [see Fig. 1(b)], the excitation spectrum of this model was predicted to be governed by a complex symmetry which is described by a quantum integrable field theory with the E_8 symmetry (an exceptional simple Lie algebra of rank 8) [12], which, however, is rarely explored experimentally. An analytical solution of the E_8 excitation spectrum delivered exactly eight particles ($\mathbf{m}_1$ to $\mathbf{m}_8$), the existence of which is uniquely determined by the specific ratios of their masses (Table I) with the lowest mass scaling with the perturbative longitudinal field; i.e., $\mathbf{m}_1 \propto |B_\parallel|^{8/15}$ [12]. Further analysis on the dynamic characteristics of the eight particles showed that the single-particle spectral weight decreases monotonically and drastically with increasing energy [Fig. 1(a)] [13,14]. Despite the apparent simplicity of the spin Hamiltonian in Eq. (1), an experimental realization of the E_8 spectrum, however, is very difficult, because several crucial criteria must be simultaneously fulfilled: one-dimensionality of spin interactions, strong Ising anisotropy, and a perturbative longitudinal field.

In this work, we use high-resolution terahertz (THz) spectroscopy to resolve E_8 particles in an antiferromagnetic Ising spin-chain material $BaCo_2V_2O_8$, where all the crucial criteria are found to be realized. By performing analytical

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E_8 Spectra of Quasi-One-Dimensional Antiferromagnet $BaCo_2V_2O_8$ under Transverse Field

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We report ^{51}V NMR and inelastic neutron scattering (INS) measurements on a quasi-1D antiferromagnet $BaCo_2V_2O_8$ under transverse field along the [010] direction. The scaling behavior of the spin-lattice relaxation rate above the Néel temperatures unveils a 1D quantum critical point (QCP) at $H_c^{1D} \approx 4.7$ T, which is masked by the 3D magnetic order. With the aid of accurate analytical analysis and numerical calculations, we show that the zone center INS spectrum at H_c^{1D} is precisely described by the pattern of the 1D quantum Ising model in a magnetic field, a class of universality described in terms of the exceptional E_8 Lie algebra. These excitations are nondiffusive over a certain field range when the system is away from the 1D QCP. Our results provide an unambiguous experimental realization of the massive E_8 phase in the compound, and open a new experimental route for exploring the dynamics of quantum integrable systems as well as physics beyond integrability.

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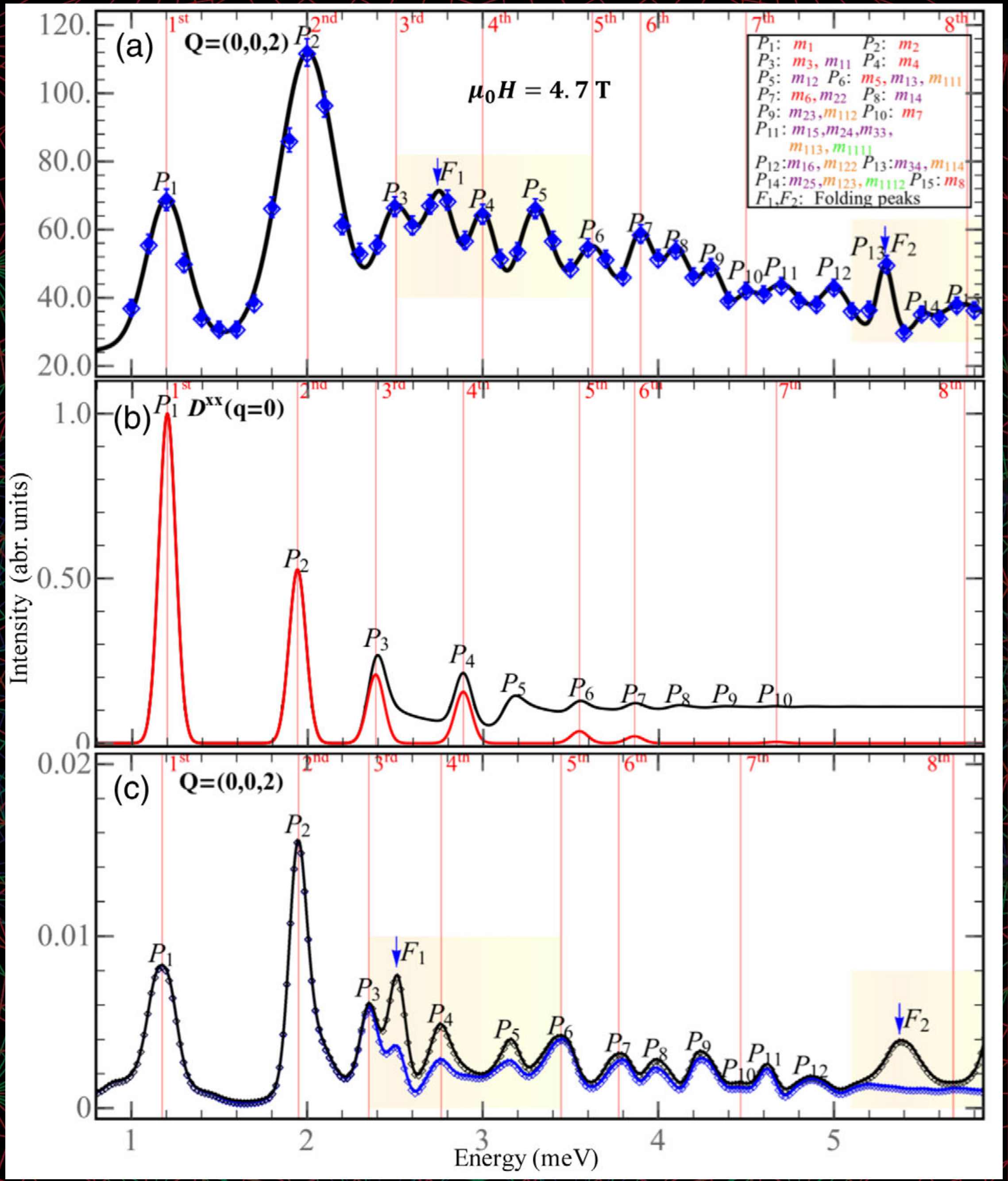
Strong fluctuations in the vicinity of quantum phase transitions can induce exotic ground states and excitations [1] such as unconventional quantum critical scalings [1,2], deconfined quantum critical points (QCPs) [3,4], and emergent enriched symmetries [5]. However, pursuing intrinsic features of these exotic states is a challenging quest, and only a few exactly solvable models provide significant insight. For example, an exotic spin liquid ground state can be characterized by the honeycomb Kitaev model [6] and stimulates serious hunting in materials [7]. Remarkably, an integrable model [8] emerges when the QCP of the paradigmatic 1D transverse-field Ising chain (TFIC) [1,2] is perturbed by a longitudinal magnetic field. The excitations of this model are beautifully characterized by the interplay of eight particles governed by the E_8 exceptional Lie algebra. This E_8 picture is a compelling pattern of the general class of the universality of the 1D TFIC once perturbed by a longitudinal magnetic field, as shown originally by Zamolodchikov [8]. Therefore, finding and exploring the E_8 physics in condensed matter systems will be a significant milestone for realizing analytically predicted emergent exotic excitations and will provide a manipulable platform for exploring quantum magnetism.

Compelling though it may be, the manifestation of this exotic E_8 physics can only be established via a dynamics study. In experiments, it is very challenging to accurately determine the location of a 1D QCP and resolve all the



(a)
Q=(0,0,2)
μ0H = 4.7 T
P1: m1 P2: m2
P3: m3, m11 P4: m4
P5: m12 P6: m5, m13, m111
P7: m6, m22 P8: m14
P9: m23, m112 P10: m7
P11: m15, m24, m33, m113, m1111
P12: m16, m122 P13: m34, m114
P14: m25, m123, m1112 P15: m8
F1, F2: Folding peaks
(b)
D^xx(q=0)
(c)
Q=(0,0,2)
Intensity (abr. units)
Energy (meV)

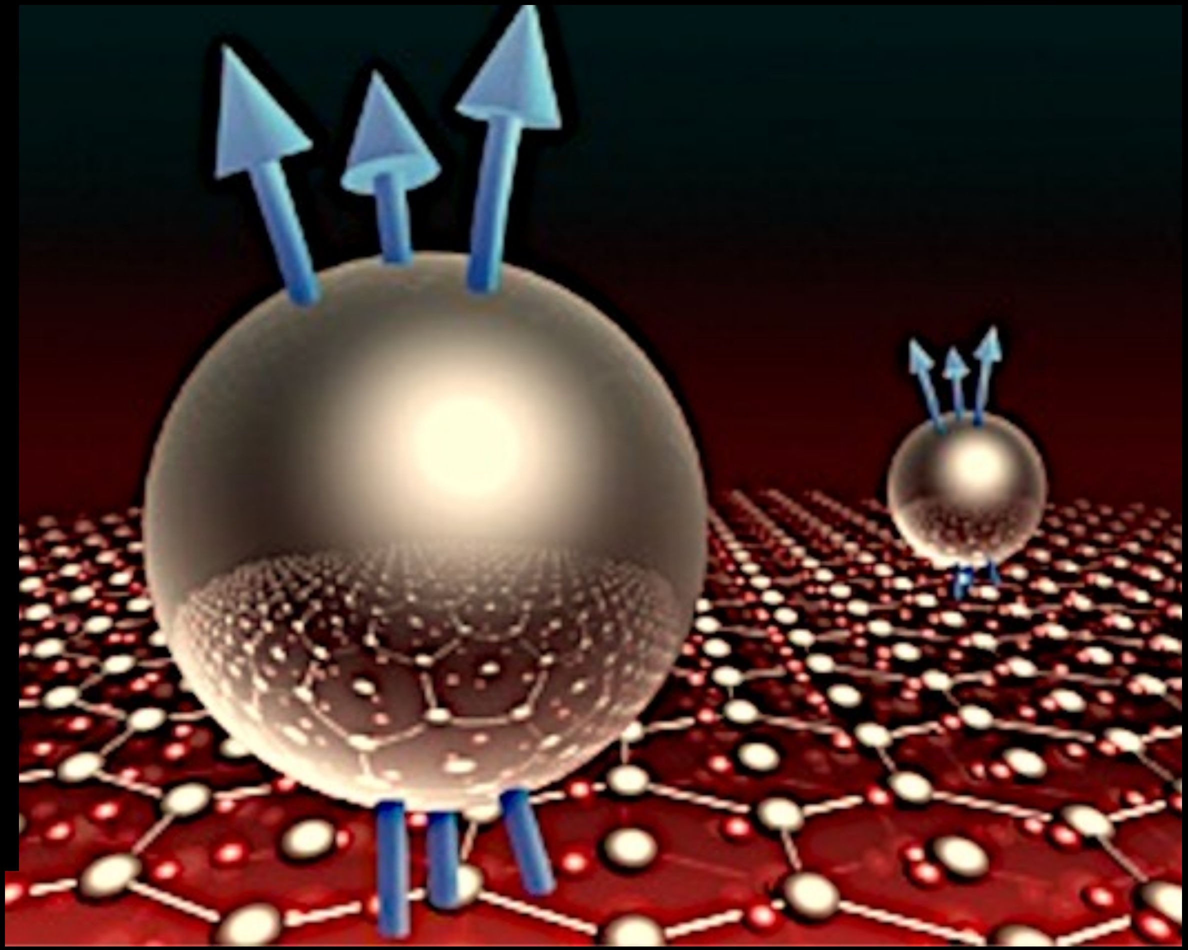
- If possible, there is a class of universality even richer and more intriguing!
- It is the one of the Tricritical Ising Model
- Beautiful symmetries: E_7, SUSY, duality, parity, etc.
- It poses an interesting and engaging challenge from the experimental point of view

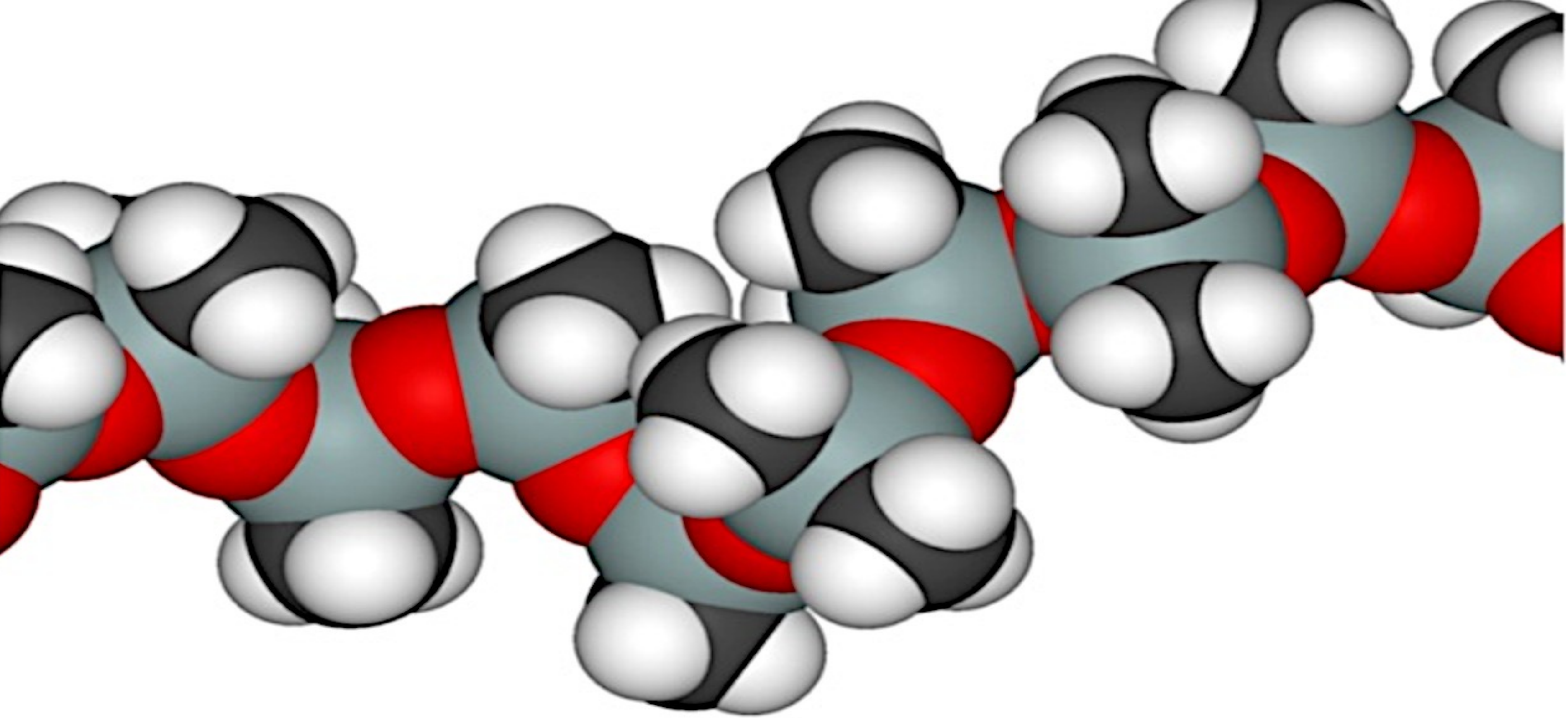
Topics of the seminar

- The class of universality of the **Tricritical Ising Model**
  - (i) Spin 1 and Blume-Capel model
  - (ii) Conformal Field Theory and its deformations
  - (iii) SUSY and E_7 symmetry
- Thermal deformation
  - (i) low-high temperature duality
  - (ii) E_7 particles, kinks and their elastic S-matrix
- Exact Form Factors and Dynamical Structure Factors
  - (i) Bootstrap equations
  - (ii) Exact solutions and operator content

The Physics of Spin 1

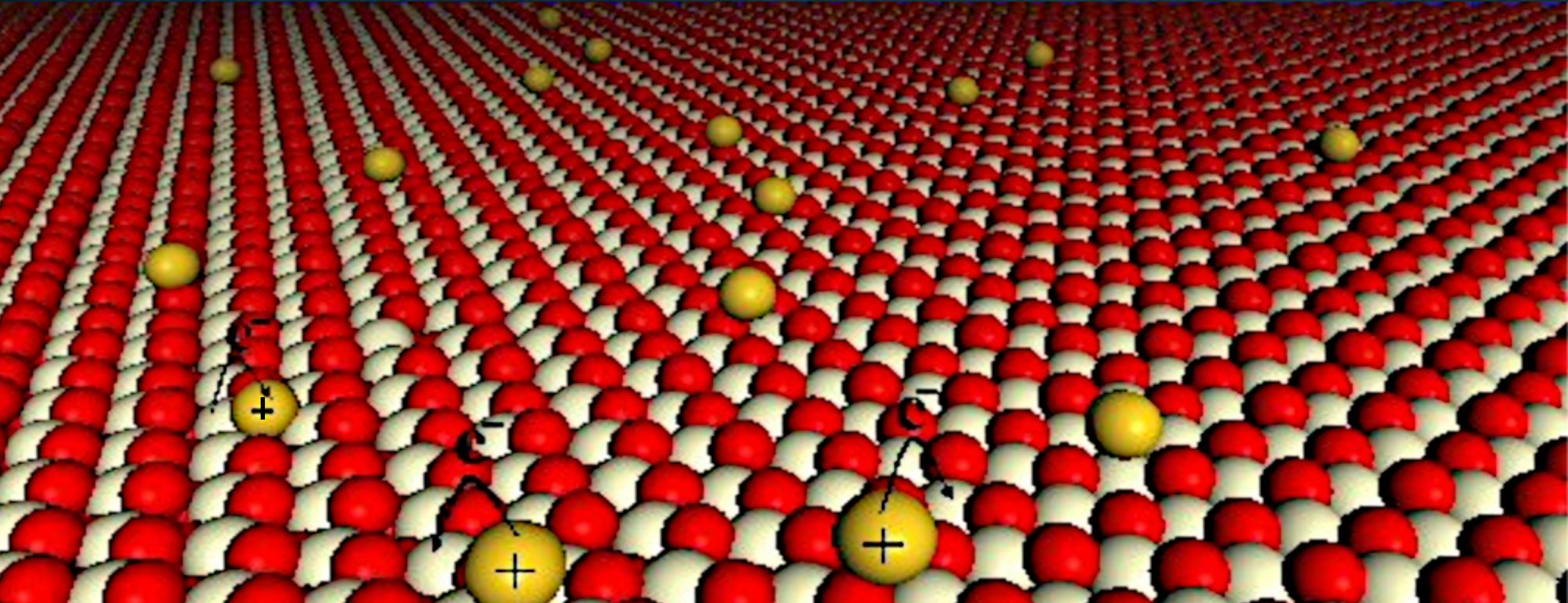
$$S_z = \{\pm 1, 0\}$$



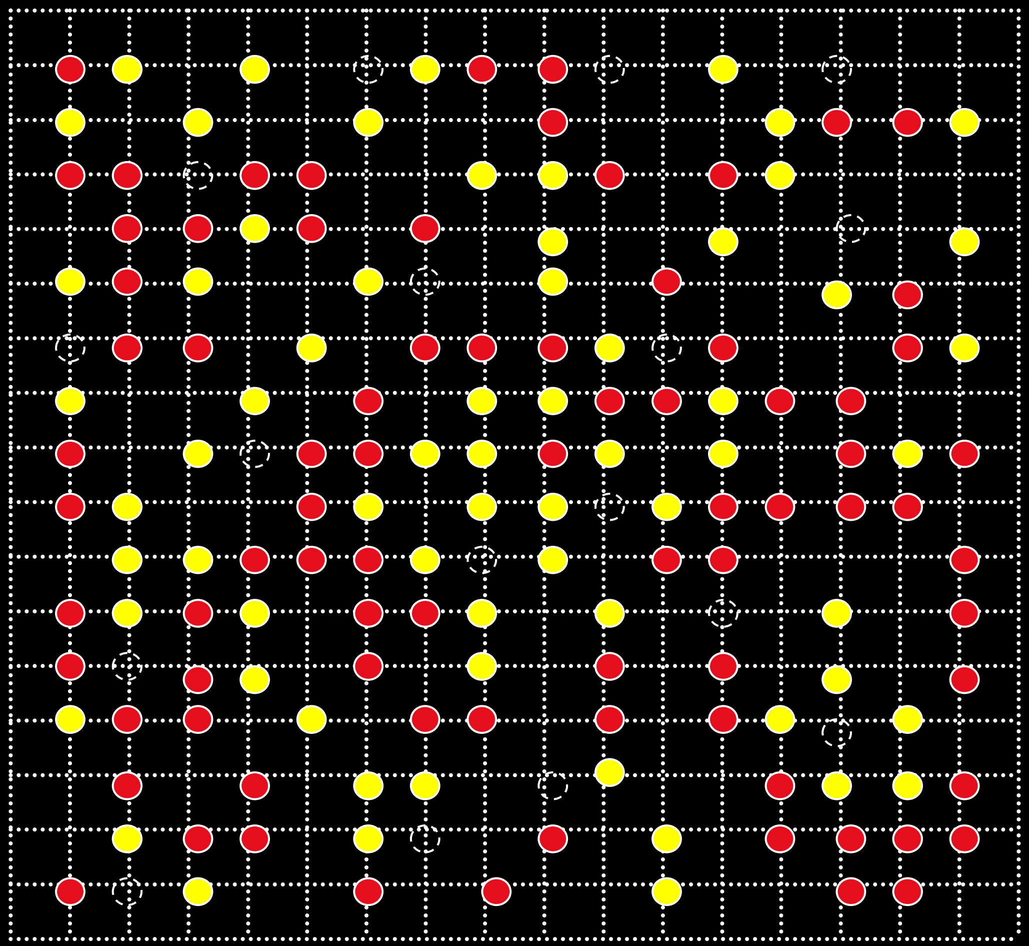


$$H = -\sum_i \left(S_z(i) S_z(i+1) - \alpha S_z^2(i) - \beta S_x(i) \right)$$

2-dimensional statistical model



Blume-Capel Model

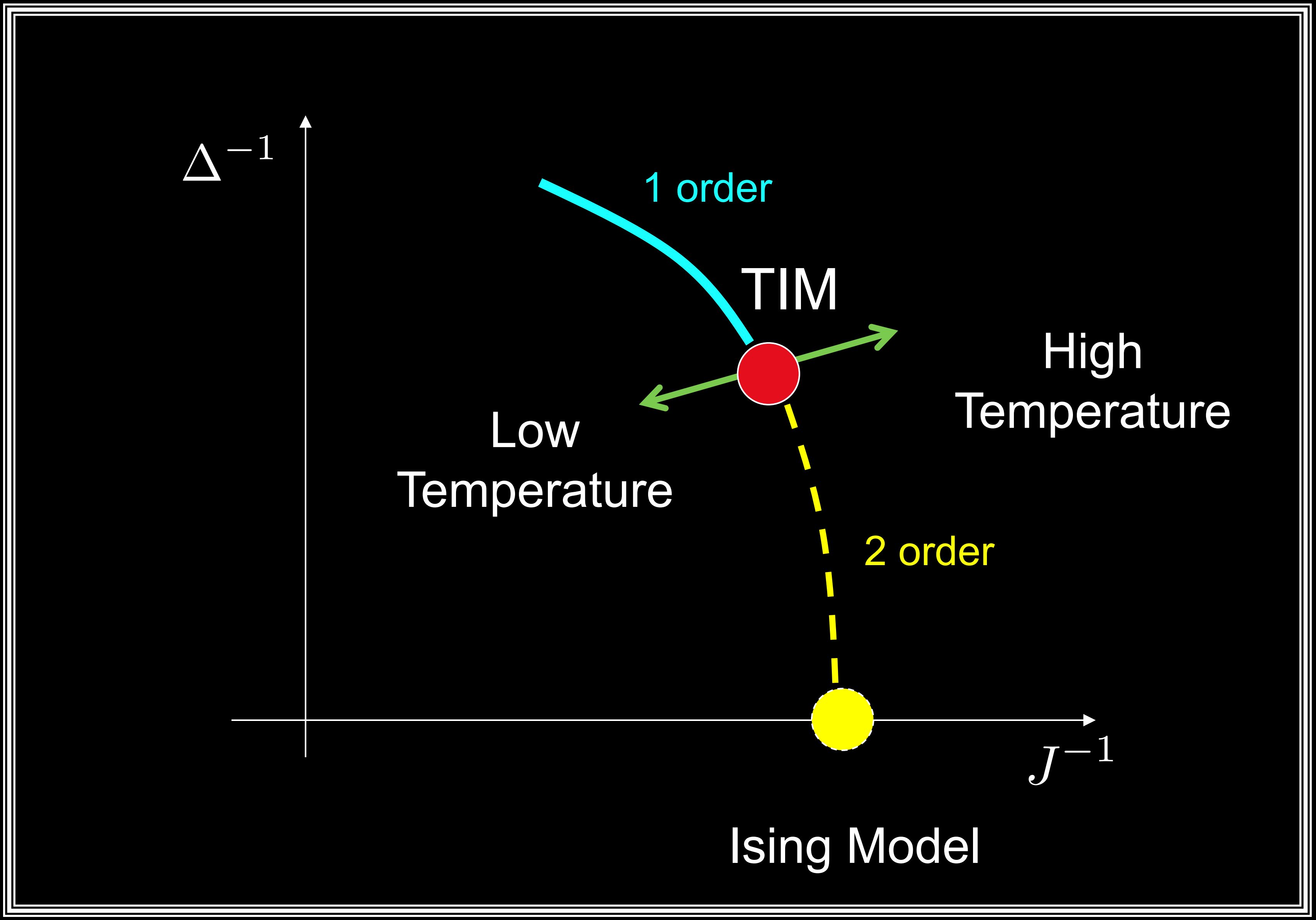


$$s_i = \{\pm 1\}$$

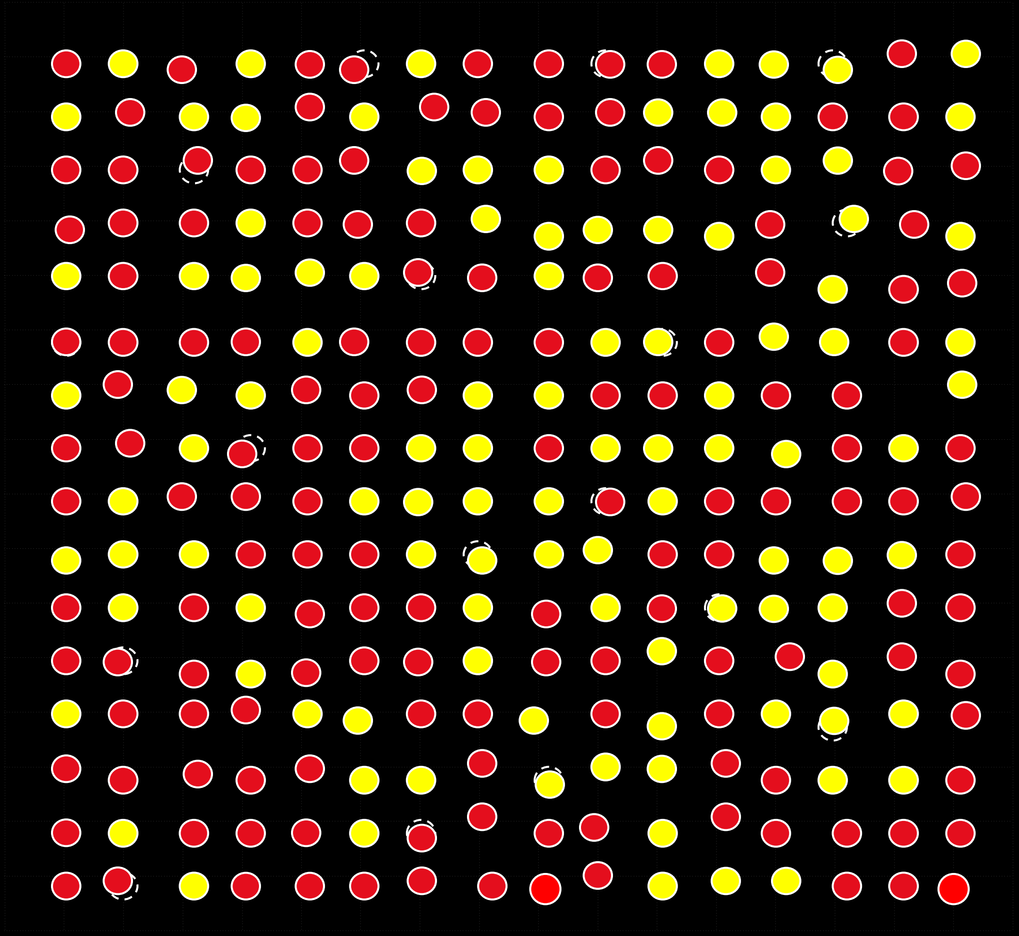
$$t_i = \{0, 1\}$$

$$H = -J \sum_{<i,j>} s_i s_j t_i t_j + \Delta \sum_i (t_i - 1) + H \sum_i s_i t_i + H' \sum_{<i,j>} (s_i t_i t_j + s_j t_i t_j)$$

Phase diagram of TIM



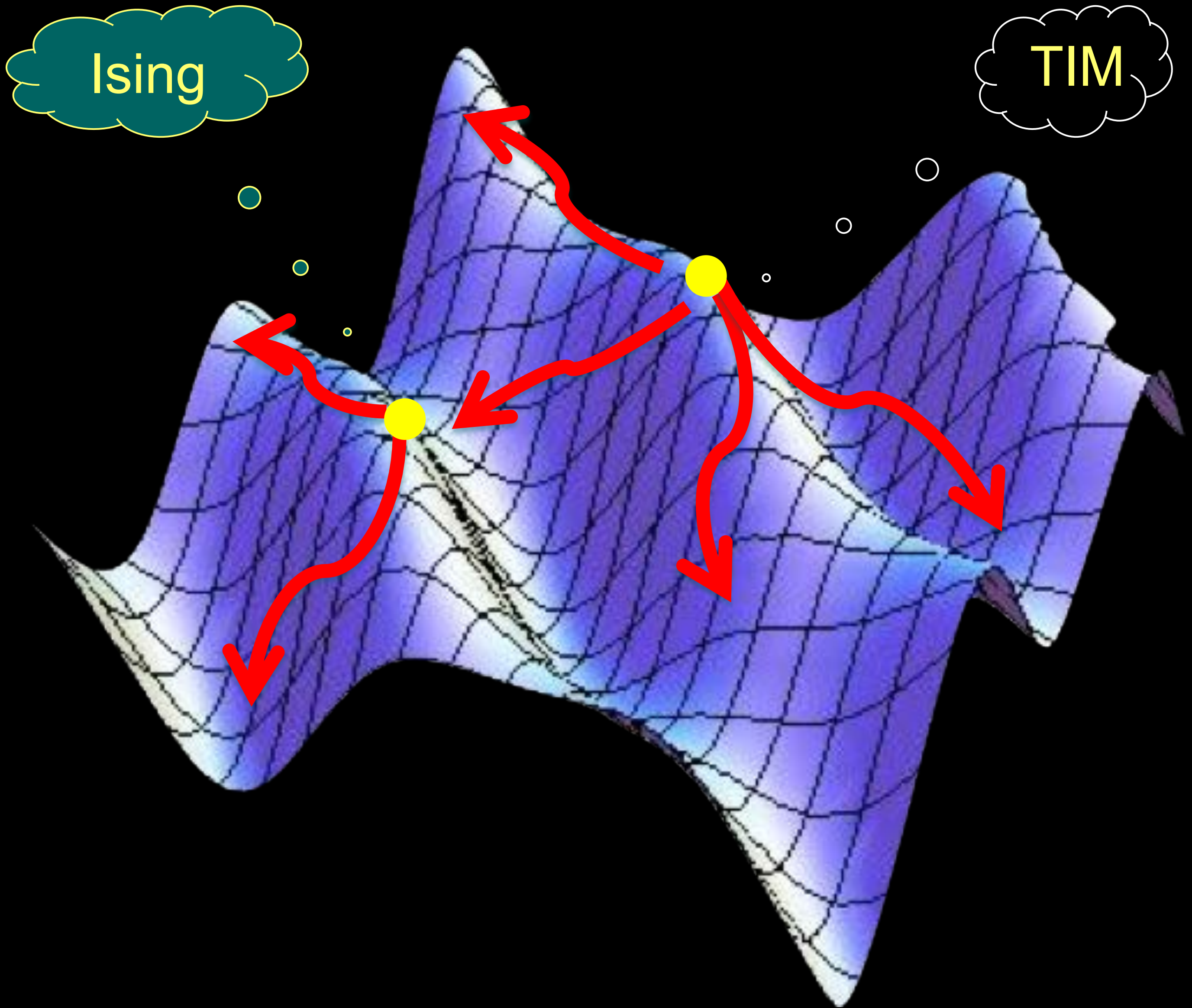
By varying the chemical potential of the vacancies



Critical Ising Model !

Space of QFT's

(BPZ)



Stress-energy tensor and Virasoro algebra

$$T(z_1)T(z_2) = \frac{c}{2(z_1-z_2)^4} + \frac{2\,T(z_2)}{(z_1-z_2)^2} + \frac{1}{z_1-z_2}\partial T(z_2)$$

Mode expansion

$$T(z) = \sum_{-\infty}^{\infty} \frac{L_n}{z^{n+2}}$$

Stress-energy tensor and Virasoro algebra

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12}n(n^2-1)\delta_{n+m,0}$$

- c identifies the classes of universality

- IR and primary operators

$$L_0 \phi_\Delta(z) = \Delta\, \phi_\Delta(z)$$

$$L_n \phi_\Delta(z) = 0\ ,\ \ n > 0$$

TIM: Operator Content from Conformal Field Theory

Second unitary minimal model. The anomalous dimensions Δ are given by the Kac table

$\frac{3}{2}$	$\frac{3}{5}$	$\frac{1}{10}$	0
$\frac{7}{16}$	$\frac{3}{80}$	$\frac{3}{80}$	$\frac{7}{16}$
0	$\frac{1}{10}$	$\frac{3}{5}$	$\frac{3}{2}$

$\sigma \Rightarrow \left(\frac{3}{80},\frac{3}{80}\right)$ $\quad \varepsilon \Rightarrow \left(\frac{1}{10},\frac{1}{10}\right)$

$\tilde{\sigma} \Rightarrow \left(\frac{7}{16},\frac{7}{16}\right)$ $\quad t \Rightarrow \left(\frac{3}{5},\frac{3}{5}\right)$

$I \Rightarrow (0,0)$ $\quad \varepsilon'' \Rightarrow \left(\frac{3}{2},\frac{3}{2}\right)$

$G \Rightarrow \left(\frac{3}{2},0\right)$ $\quad \bar{G} \Rightarrow \left(0,\frac{3}{2}\right)$

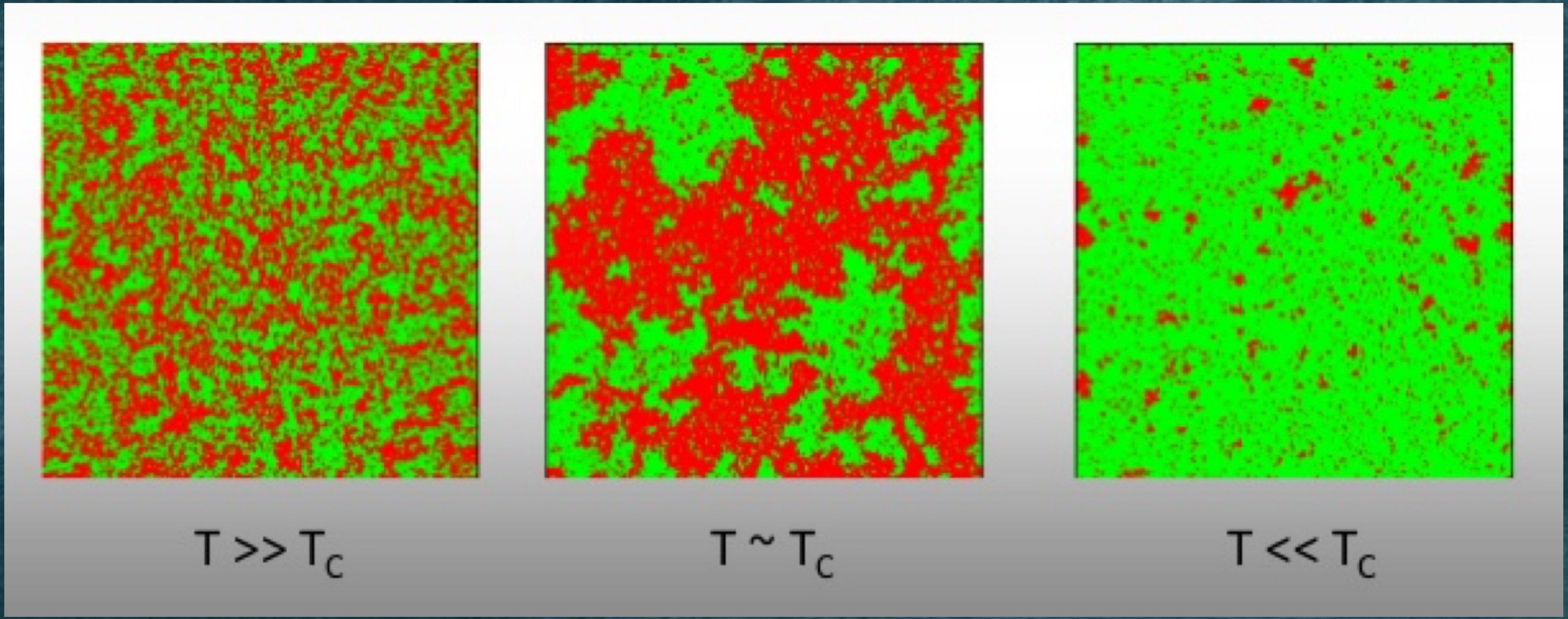
$\Psi \Rightarrow \left(\frac{3}{5},\frac{1}{10}\right)$ $\quad \bar{\Psi} \Rightarrow \left(\frac{1}{10},\frac{3}{5}\right)$

The central charge is c=7/10

Fusion Rules and Structure constants of TIM

even ∗ even $\epsilon * \epsilon = [1] + c_1\,[t]$ $t * t = [1] + c_2\,[t]$ $\epsilon * t = c_1\,[\epsilon] + c_3\,[\varepsilon'']$	$c_1 = \frac{2}{3}\sqrt{\frac{\Gamma\left(\frac{4}{5}\right)\Gamma^3\left(\frac{2}{5}\right)}{\Gamma\left(\frac{1}{5}\right)\Gamma^3\left(\frac{3}{5}\right)}}$ $c_2 = c_1$
even ∗ odd $\epsilon * \sigma' = c_4\,[\sigma]$ $\epsilon * \sigma = c_4\,[\sigma'] + c_5\,[\sigma]$ $t * \sigma' = c_6\,[\sigma]$ $t * \sigma = c_6\,[\sigma'] + c_7\,[\sigma]$	$c_3 = \frac{3}{7}$ $c_4 = \frac{1}{2}$ $c_5 = \frac{3}{2}c_1$ $c_6 = \frac{3}{4}$ $c_7 = \frac{1}{4}c_1$ $c_8 = \frac{7}{8}$ $c_9 = \frac{1}{56}$
odd ∗ odd $\sigma' * \sigma' = [1] + c_8\,[\varepsilon'']$ $\sigma' * \sigma = c_4\,[\epsilon] + c_6\,[t]$ $\sigma * \sigma = [1] + c_5\,[\epsilon] + c_7\,[t] + c_9\,[\varepsilon'']$	

Duality: order and disorder operators



Kramers-Wannier (1941)
J. Frohlich et al. (2006)

- In the TIM, each order operator σ is accompanied by its dual disorder operator τ, of equal conformal dimension
- As in the Ising Model, this is a consequence of the presence of fermionic fields!

Fermionic fields and Supersymmetry

$$G(z_1)G(z_2) = \frac{2\,c}{3(z_1 - z_2)^3} + \frac{2\,T(z_2)}{z_1 - z_2} + \cdots$$

Mode expansion

$$G(z) = \sum_{-\infty}^{\infty} \frac{G_n}{z^{n+3/2}} \qquad n = \begin{cases} k + \frac{1}{2} & , \quad NS \\ k & , \quad R \end{cases}$$

Fermionic fields and Supersymmetry

$$\{G_n, G_m\} = 2\,L_{n+m} + \frac{c}{3}\left(n^2 - \frac{1}{4}\right)\delta_{n+m,0}$$

$$[L_n, G_m] = (n/2 - m)\,G_{n+m}$$

IR and primary operators (NS and R)

- NS Sector: Superfield

$$\Phi(z,\overline{z},\theta,\overline{\theta}) = \epsilon(z,\overline{z}) + \theta\,\overline{\psi}(z,\overline{z}) + \overline{\theta}\,\psi(z,\overline{z}) + \theta\overline{\theta}\,t(z,\overline{z})$$

- R Sector: Ramond fields $(\sigma, \tilde{\sigma})$

$$G_0^2 = L_0 - \frac{c}{24} \Rightarrow \begin{array}{l} G_0\,\sigma \propto \tau \\ G_0\,\tau \propto \sigma \end{array}$$

Doublets!!

Fermionic fields and Supersymmetry

$$\{G_n, G_m\} = 2\,L_{n+m} + \frac{c}{3}\left(n^2 - \frac{1}{4}\right)\delta_{n+m,0}$$

$$[L_n, G_m] = (n/2 - m)\,G_{n+m}$$

IR and primary operators (NS and R)

- NS Sector: Superfield

$$\Phi(z,\overline{z},\theta,\overline{\theta}) = \epsilon(z,\overline{z}) + \theta\,\overline{\psi}(z,\overline{z}) + \overline{\theta}\,\psi(z,\overline{z}) + \theta\overline{\theta}\,t(z,\overline{z})$$

- R Sector: Ramond fields $(\sigma, \tilde{\sigma})$

$$G_0^2 = L_0 - \frac{c}{24} \quad \Longrightarrow \quad G_0\tilde{\sigma} \propto \tilde{\tau}, \quad G_0\tilde{\tau} \propto \tilde{\sigma}$$

Doublets!!

Z_2 Symmetries

Spin symmetry

$$\sigma \Rightarrow -\sigma$$

$$\varepsilon \Rightarrow \varepsilon$$

$$\tilde{\sigma} \Rightarrow -\tilde{\sigma}$$

$$t \Rightarrow t$$

Kramers-Wannier duality

$$\sigma \Rightarrow \tau$$

$$\varepsilon \Rightarrow -\varepsilon$$

$$\tilde{\sigma} \Rightarrow \tilde{\tau}$$

$$t \Rightarrow t$$

Hidden symmetries

Equivalent Coset constructions of the TIM

- $SU(2)$ $$\frac{SU(2)_4 \times SU(2)_1}{SU(2)_5}$$
- E_7 $$\frac{(E_7)_1 \times (E_7)_1}{(E_7)_2}$$
- SUSY $$\frac{SU(2)_1 \times SU(2)_2}{SU(2)_3}$$

Landau-Ginzburg description

$$\frac{(SU(2))_4 \times (SU(2))_1}{(SU(2))_5}$$

$$\sigma \equiv \; :\Phi:$$
$$\epsilon \equiv \; :\Phi^2:$$
$$\tilde{\sigma} \equiv \; :\Phi^3:$$
$$t \equiv \; :\Phi^4:$$

$$L = \frac{1}{2}\left(\partial_\mu \Phi\right)^2 + V(\Phi)$$

$$V(\Phi) = g_1\Phi + g_2\Phi^2 + g_3\Phi^3 + g_4\Phi^4 + \Phi^6$$

Nature of the QFT of the relevant deformations of TIM

$$V(\Phi) = g_1 \Phi + g_2 \Phi^2 + g_3 \Phi^3 + g_4 \Phi^4 + \Phi^6$$

g_1	Particles	Non-Integrable
g_2^+	High T (particles)	Integrable E_7
g_2^-	Low T (kinks and bound states thereof)	
g_3	Asymmetrical kinks	Integrable
g_4^+	Massless particles	Integrable SUSY
g_4^-	Kinks