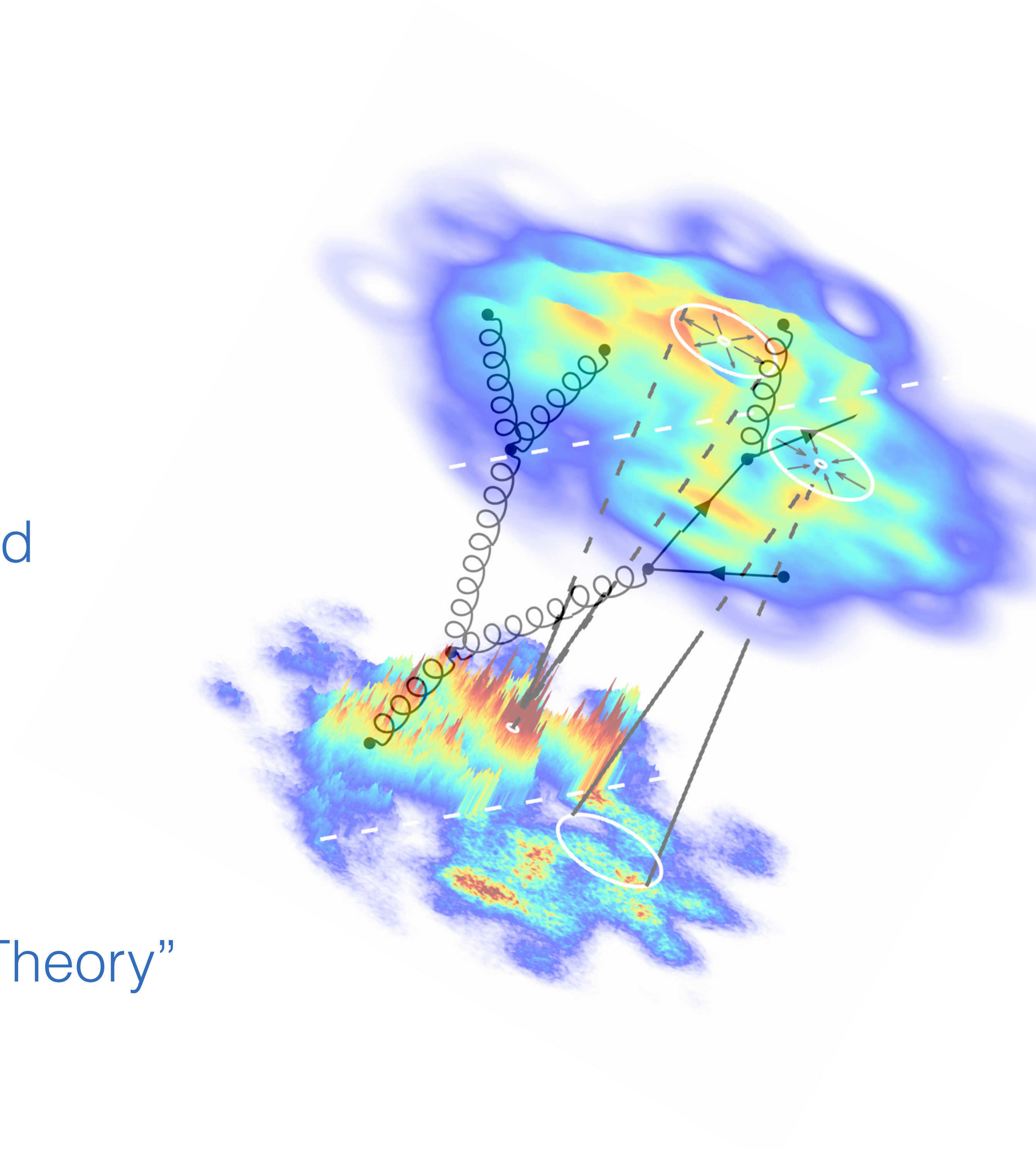
Turbulence in High-Energy Physics



Sören Schlichting | Universität Bielefeld

Reviews:
SS, Teaney, Ann.Rev.Nucl.Part.Sci. 69 (2019) 447-476
Berges, Heller, Mazeliauskas, Venugopalan arXiv:*2005.12299*

Workshop on “Turbulence and Field Theory”
The Graduate Center @ CUNY
Mar 2021 (Online)

CRC-TR 211
Strong-interaction matter under extreme conditions

UNIVERSITÄT BIELEFELD

Outline

1. Introduction & Motivation
2. Turbulence in non-abelian gauge theories
3. Turbulence in scalar field theories
4. Conclusions & Outlook

Turbulence in HEP

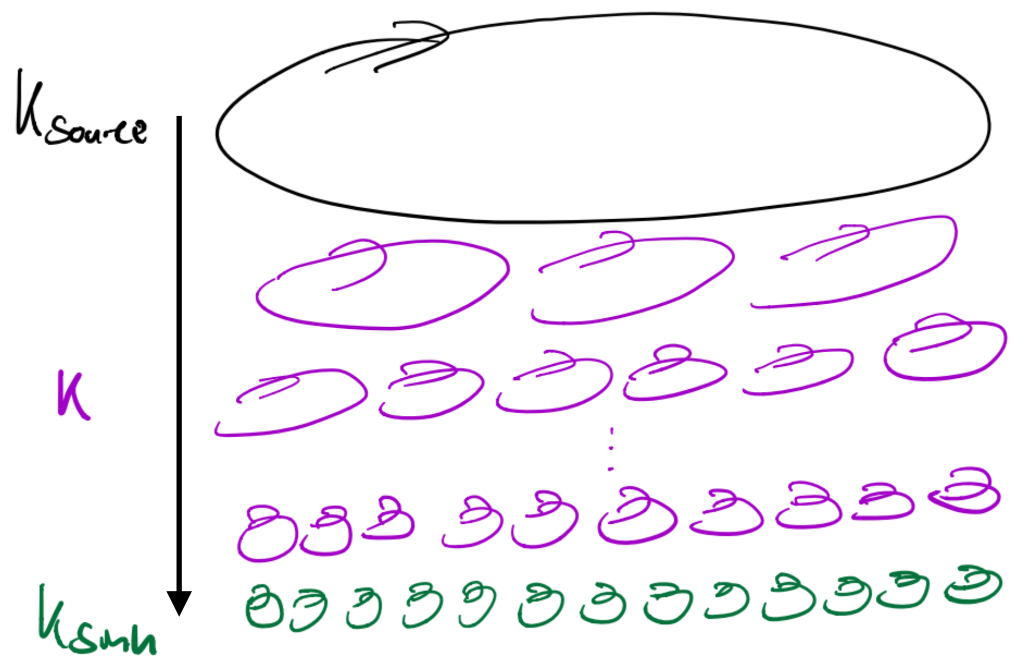
What is turbulence? —
non-equilibrium dynamics associated with transport of a conserved quantity across a large separation of scales

-> universal behavior

Different manifestations in different physical systems

stationary vs. decaying turbulence

energy injection (source)



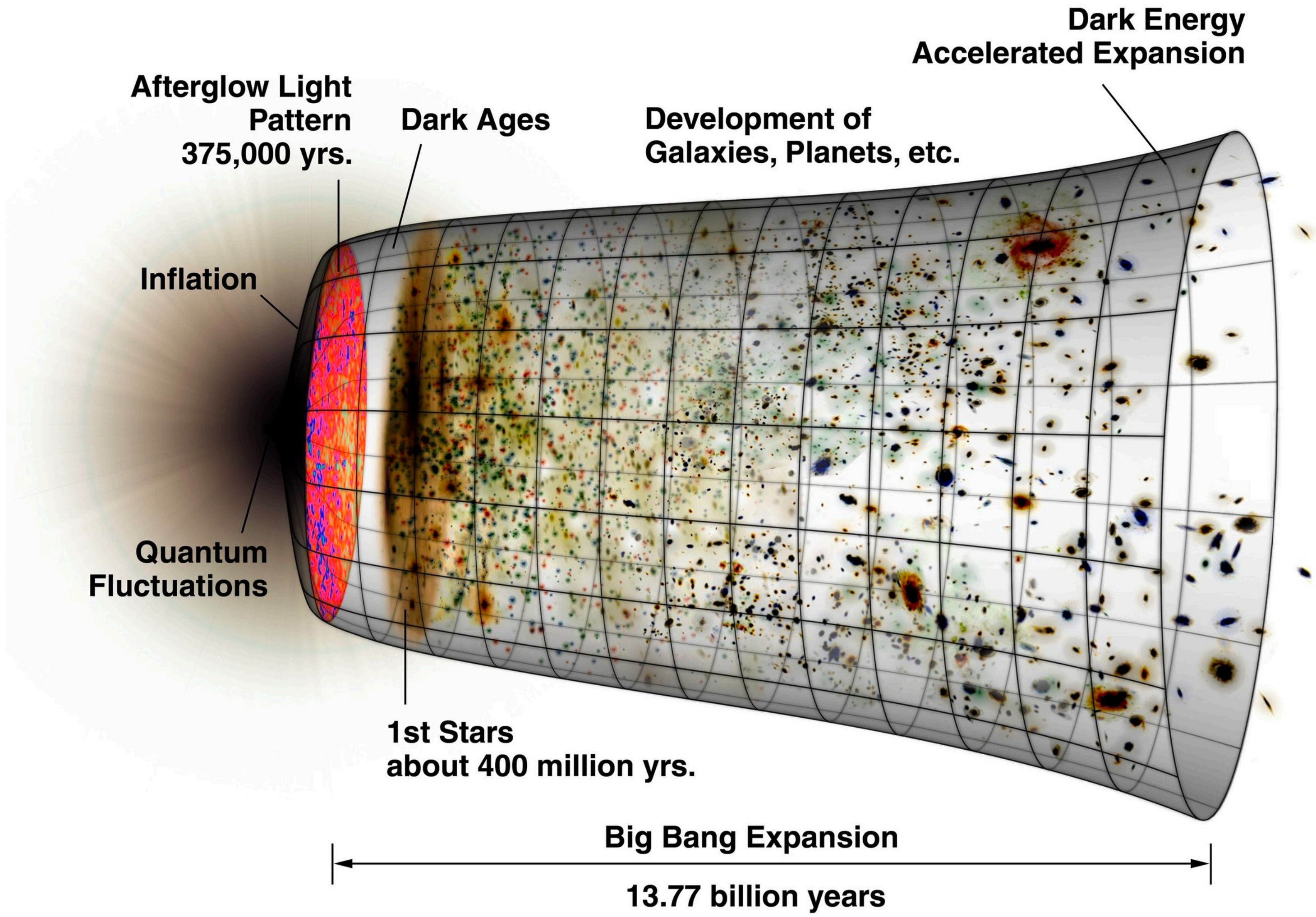
viscous dissipation (sink)

Non-equilibrium system ubiquitous in nature with many systems in HEP (Early Universe,Heavy-Ion Collisions,…) exhibiting a large separation of scales

Since HEP systems are typically closed should expect decaying turbulence rather than stationary turbulence

Far-from-equilibrium dynamics in HEP

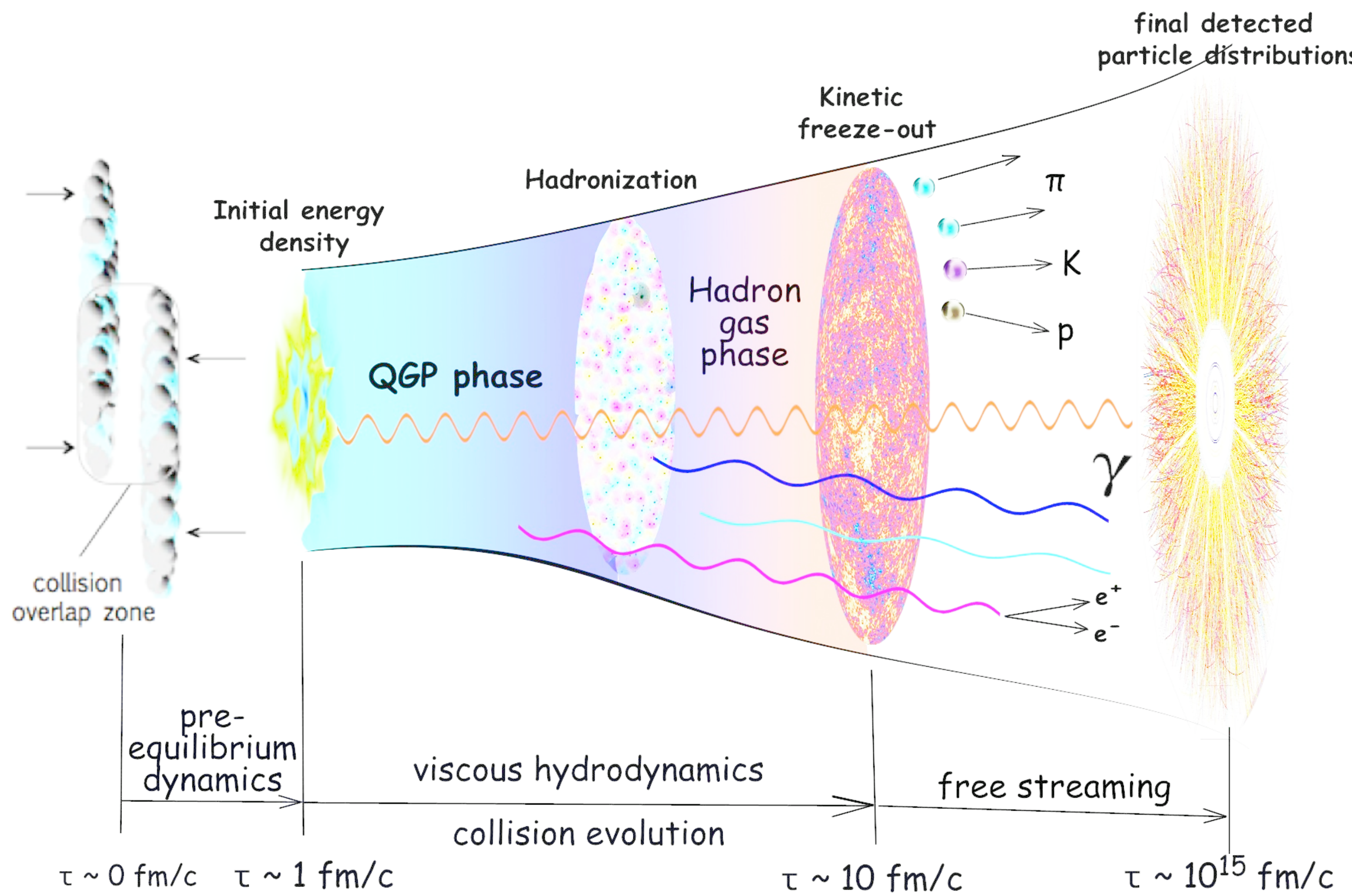
Early Universe:



How is Standard Model Matter produced and equilibrated between end of Inflation and Big Bang Nucleosynthesis (BBN)?

Far-from-equilibrium dynamics in HEP

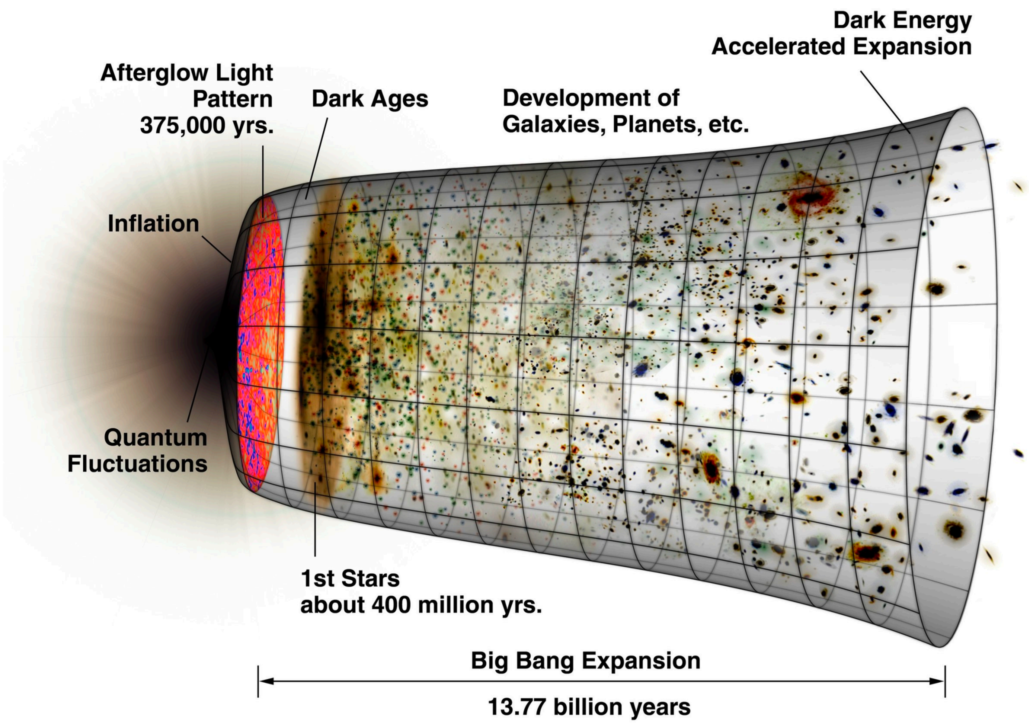
High-Energy Heavy-Ion Collisions (HICs):



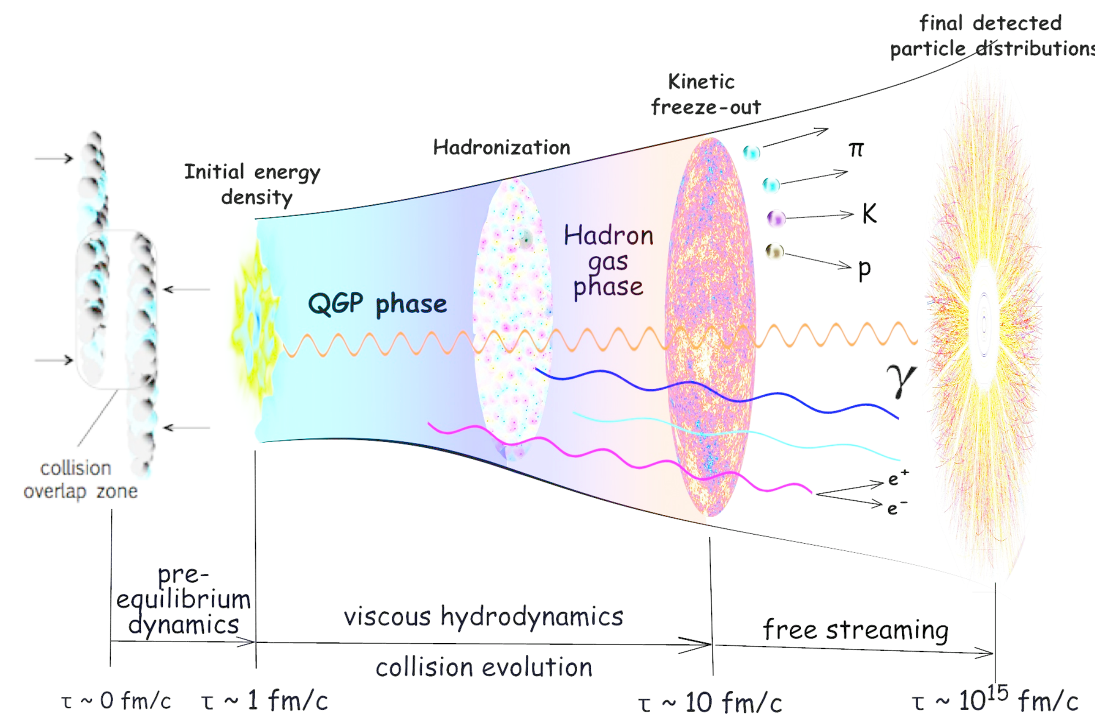
How is a new state of matter, the Quark-Gluon Plasma (QGP), created from dynamics of "primordial" far-from equilibrium plasma created in the collision?

Disclaimer

Early Universe:



Heavy-Ion Collisions:



Will not discuss details relevant to Thermalization of the Early Universe or Heavy-Ion Collisions, but instead use them as motivation for simpler examples which are better understood an have a clear connection to turbulence

Non-equilibrium QFT

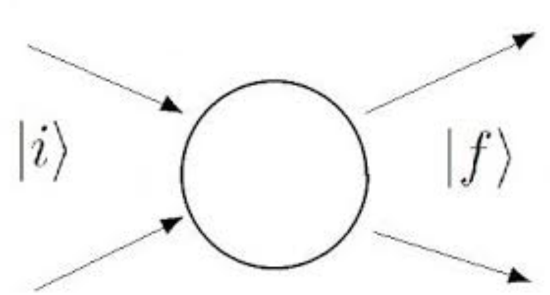
High-Energy Physics systems described by Quantum Field Theories

Generally there is no exact way to study non-equilibrium dynamics in an interacting quantum field theory

Weak coupling limit of QFT allows for description of non-equilibrium dynamics based on

Kinetic theory:

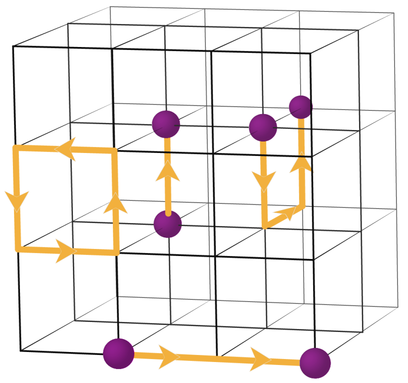
perturbative description in terms of weakly interacting quasi-particles



Classical-statistical field theory

non-perturbative description of bosonic quantum fields in terms of classical fields

numerical solution of lattice discretized EOMs



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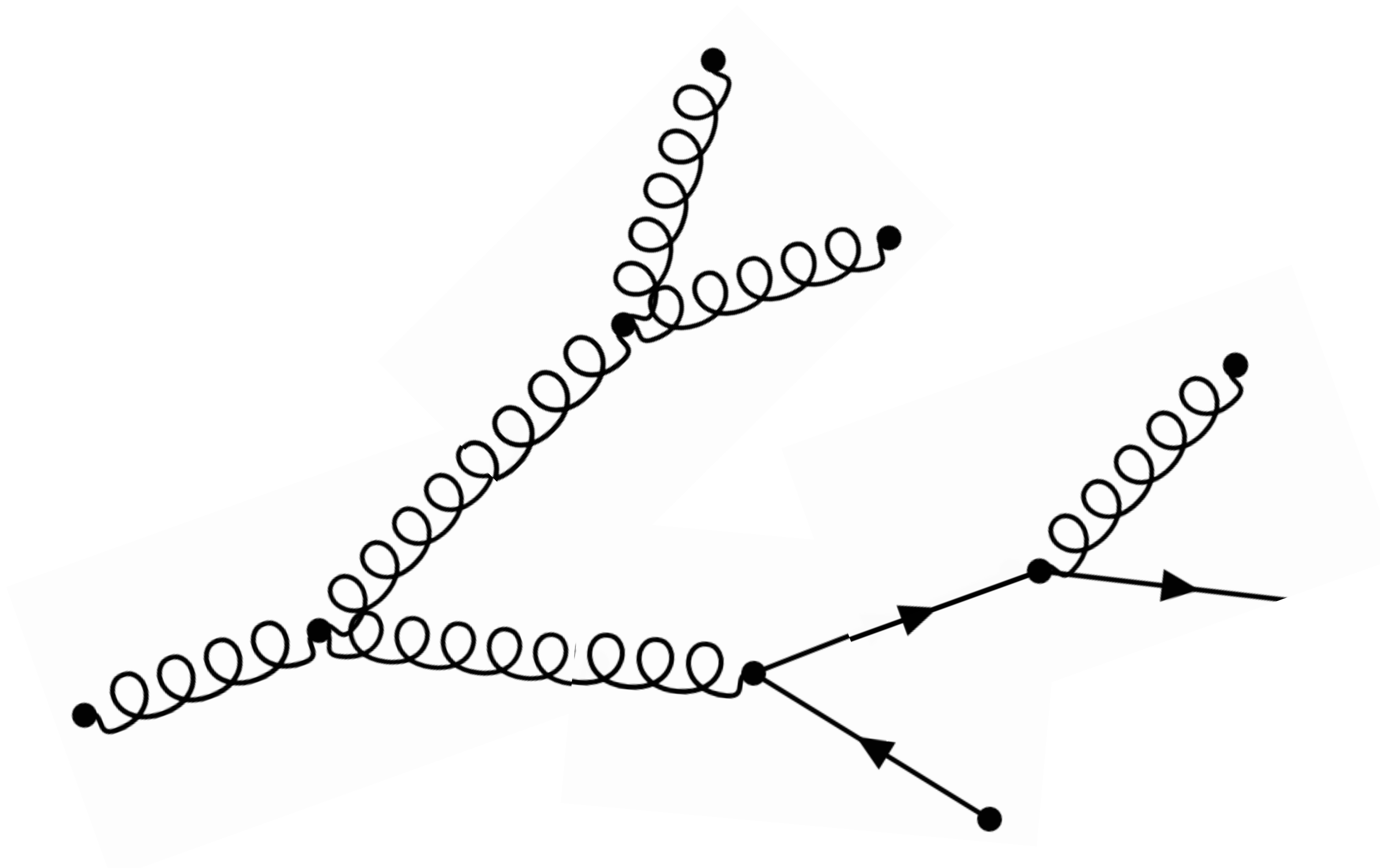


Turbulence in non-abelian gauge theories

Non-equilibrium QCD

Strong interactions described by Quantum Chromodynamics (QCD)

$$\mathcal{L}_{QCD} = \sum_{f}^{N_f} \bar{q}_f(\gamma^\mu D_\mu + m_f)q_f - \frac{1}{2}\mathrm{tr}F_{\mu\nu}^2$$

fundamental dof's are self-interacting gauge bosons (gluons) and light & heavy Dirac fermions (quarks)

Non-perturbative features (confinement,chiral symmetry breaking, …) at low energy scales <1 GeV, but asymptotically free at high energies

Dynamics of QCD at LO* described by relativistic Boltzmann equation

Arnold,Moore,Yaffe JHEP 0301 (2003) 030

$$p^\mu \partial_\mu f(x,p) = \mathcal{C}_{2\leftrightarrow 2}[f] + C_{1\leftrightarrow 2}[f]$$

considerations apply to non-abelian SU(N) plasmas

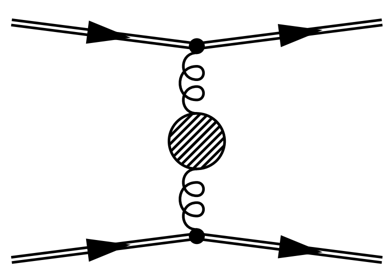
*Note that expansion is in g rather than $\alpha_s = g^2/4\pi$

Non-equilibrium QCD

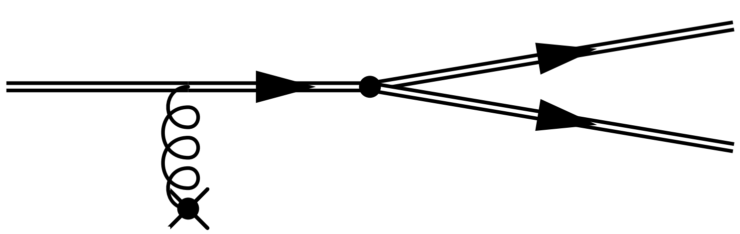
Characteristic features of effective kinetic theory of QCD

$$p^\mu \partial_\mu f(x,p) = \mathcal{C}_{2\leftrightarrow 2}[f] + C_{1\leftrightarrow 2}[f]$$

- ultra-relativistic massless quasi-particles (g,u,ubar,d,dbar,s,sbar)
- scale invariant interactions
- elastic (2<->2) & in-elastic (1<->2) proccesses at the same order



elast. 2<->2 scattering screened by Debye mass



collinear 1<->2 Bremsstrahlung
incl. Landau-Pomeranchuk-Migdal (LPM) efffect
via eff. vertex re-summation

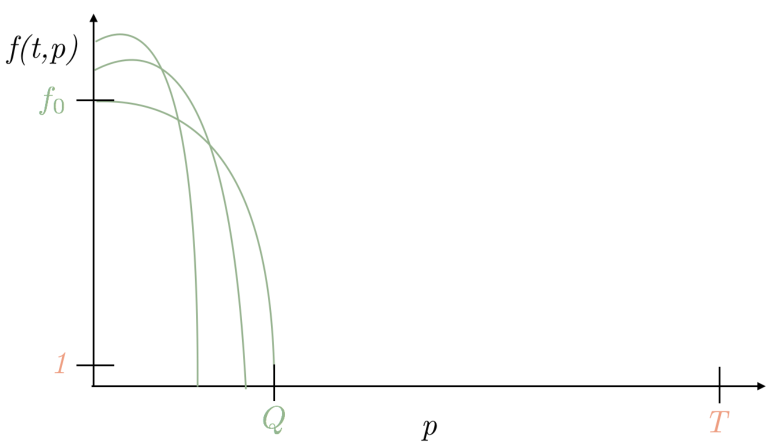
Solve numerically as integro-differential equation, with in-medium matrix elements for 2<->2 and 1<->2 processes self-consistently determined

Turbulence in QCD plasmas

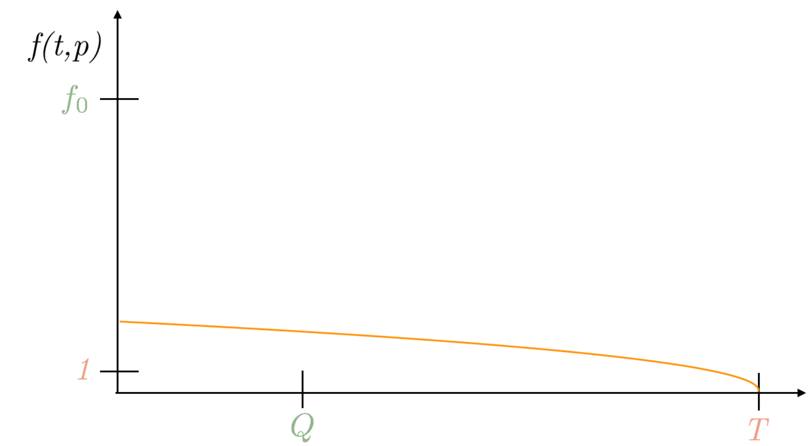
Will not address the complex problem of thermalization in HICs but instead discuss thermalization of *homogenous & isotropic QCD plasmas*

c.f. (weak-) wave-turbulence in statistically homogenous & isotropic media

Far-from equilibrium initial state



Thermal equilibrium



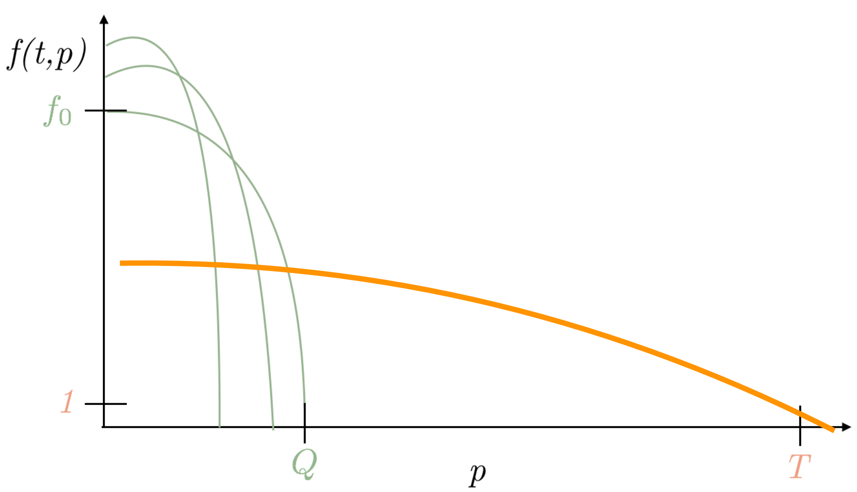
Equilibration of the system requires transport of conserved quantities across a large separation of scales

Since system closed final equilibrium state is determined by conserved quantities of the system — energy density: e, valence charge: Δn_f

Turbulence in QCD plasmas

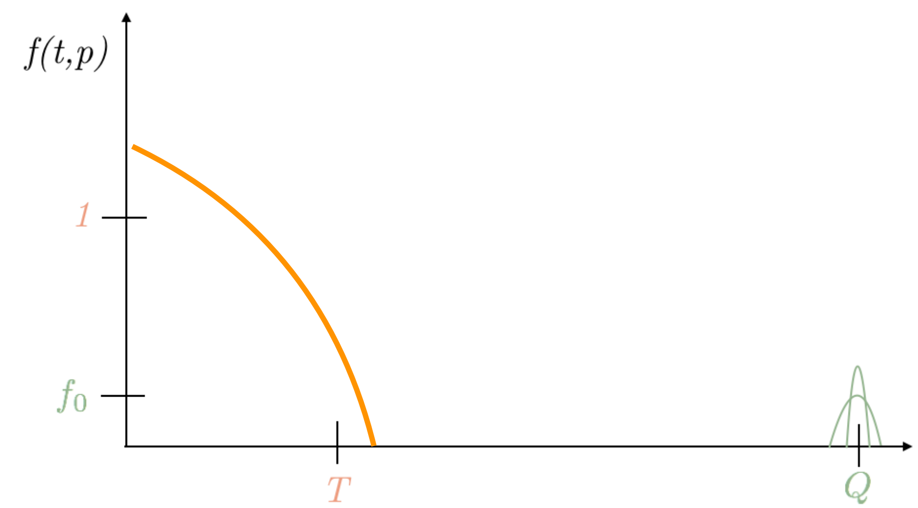
Distinguish between two qualitiatively different far from-equilibrium scenarios

Over-occupied system



Energy carried by large number of low energy dof’s <p> << T (e.g. due to instabilities)

Under-occupied system



Energy carried by small number of high energy dof’s <p> >> T (e.g. high-energy jets)

for which basic thermalization mechanisms have been worked out

Kurkela, Moore *JHEP* 12 (2011) 044

Over-occupied QCD plasmas

Classical-statistical simulations of non-equilibrium dynamics

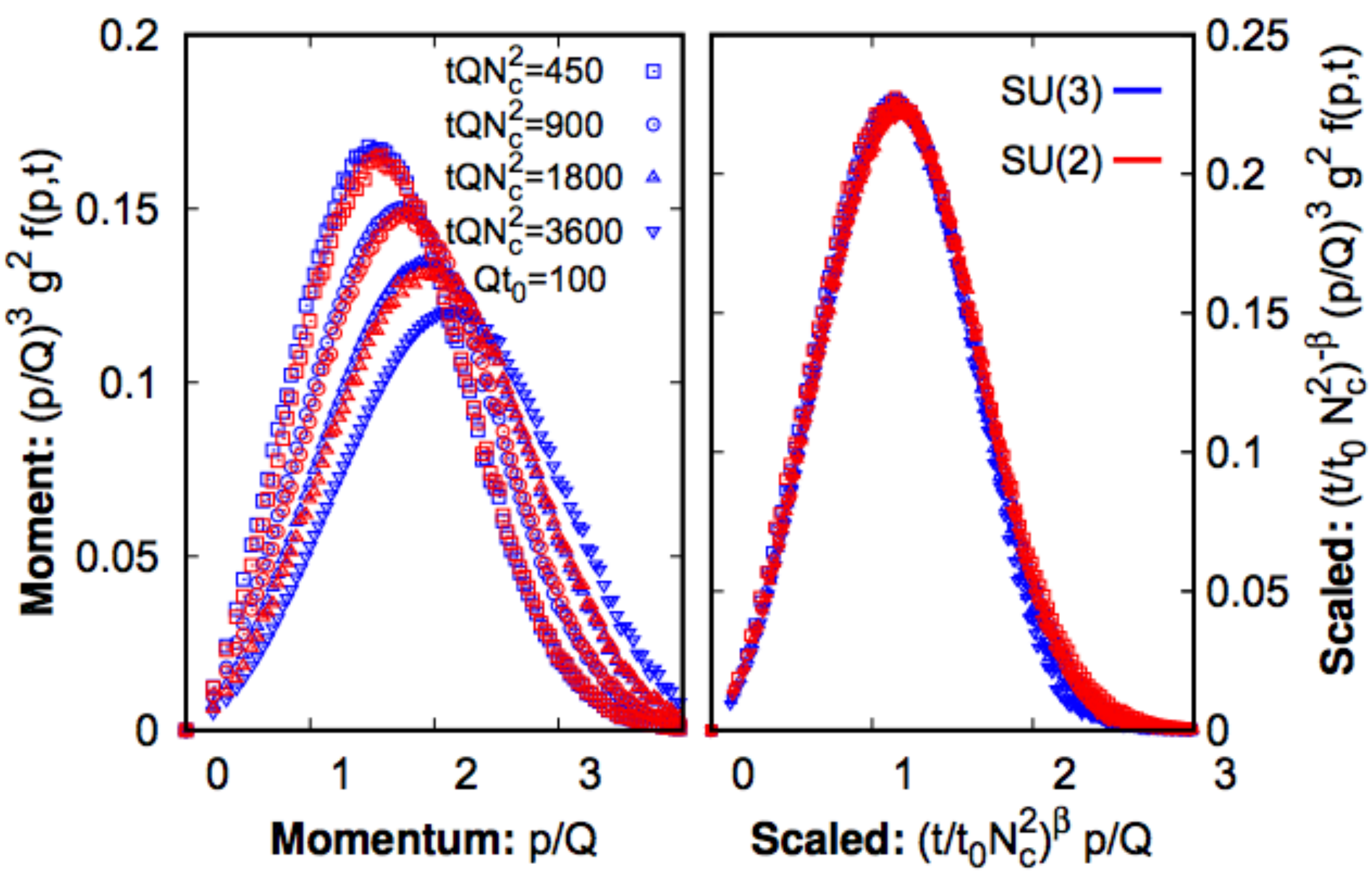
Early time dynamics:

Strongly depends on the initial conditions and can be essentially non-perturbative

Intermediate times:

Evolution becomes insensitive to initial conditions and proceeds via a self-similar ultra-violet cascade

$$f_g(t,p) = t^{\alpha} f_g^S(t^{\beta} p)$$



SS PRD 86 (2012); Berges, Boguslavski, SS, Venugopalan *PRD* 89 (2014) 11; Berges, Mace SS PRL 118 (2017) 19;

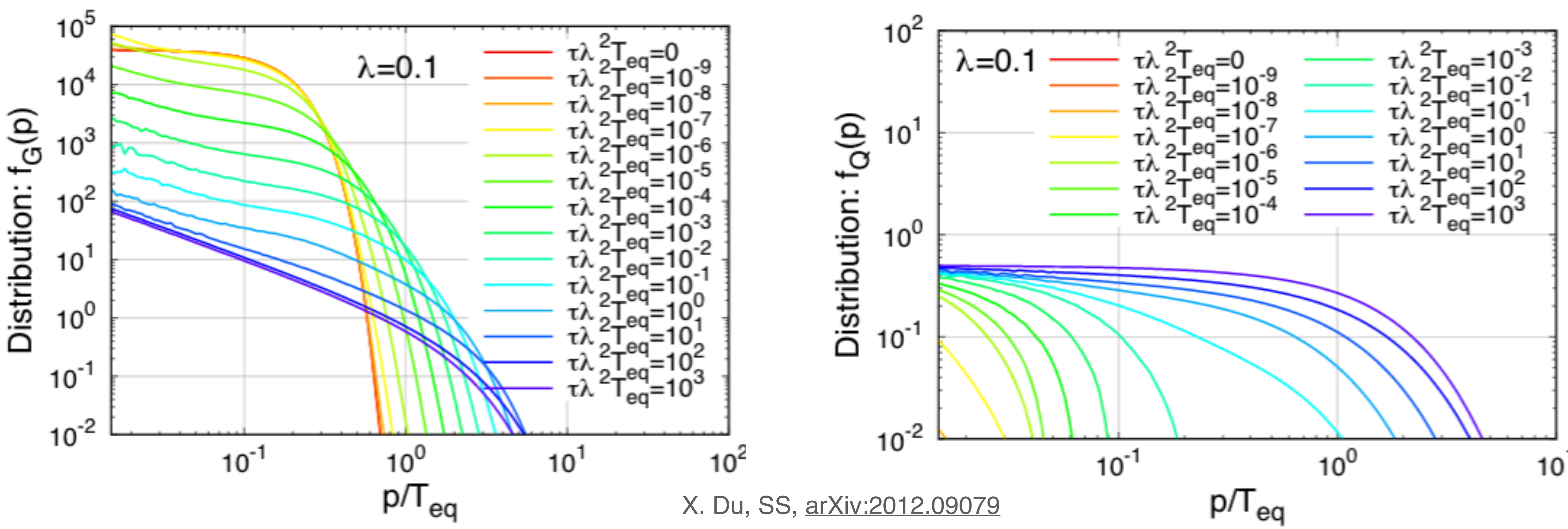
Dynamics can be entirely described in terms of

- scaling exponents α=-4/7 β=-1/7
- stationary scaling functions $f^S_g(x)$

Over-occupied QCD plasmas

Effective kinetic description reproduces class. statistical results

SS Phys.Rev.D 86 (2012); Abrao York, Kurkela, Lu, Moore *Phys.Rev.D* 89 (2014) 7;
Berges, Boguslavski, SS, Venugopalan *Phys.Rev.D* 89 (2014) 11; Berges, Mazeliauskas Phys.Rev.Lett. 122 (2019)



X. Du, SS, arXiv:2012.09079

Self-similar evolution of gluon distribution $f_G(t,p)$ associated with decaying turbulence

$$f_g(t,p) = t^{\alpha} f_g^S(t^{\beta} p)$$

Quarks are sub-dominant and simply follow gluon distribution

Equilibration occurs when energy transport to UV is accomplished

Scaling analysis

Scaling exponents α,β determined by standard scaling analysis

$$\frac{\partial f(t,\mathbf{p})}{\partial t} = C[f](t,\mathbf{p})$$

Search for self-similar scaling solution

$$f(p,t) = t^{\alpha} f_S(t^{\beta} p)$$

Scaling behavior of the collision integral

$$\xrightarrow[(f \gg 1)]{\text{scale invariance}} C[f](p,t) = t^{\mu} C[f_S](t^{\beta} p)$$

-> Boltzmann equation can be decomposed into

$$[\alpha + \beta\, \mathbf{p} \cdot \nabla_{\mathbf{p}}] f_S(\mathbf{p}) = C[f_S](1,\mathbf{p}),$$

$$\alpha - 1 = \mu(\alpha, \beta)$$

time independent fixed-point condition

scaling relation

c.f. Micha, Tkachev PRD 70 (2004) 043538 (Cosmology); Abrao York, Kurkela, Lu, Moore PRD 89 (2014) 7;
Berges, Boguslavski, SS, Venugopalan PRD 89 (2014) 11; (QCD)

Scaling analysis

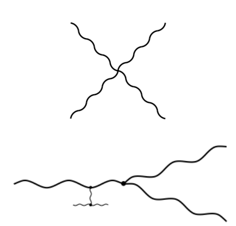
Dynamical scaling exponents *α, β* are uniquely determined by

Scaling of the collision integral **+** ***Conservation laws***

$$\alpha - 1 = \mu(\alpha, \beta)$$

$$\alpha = \beta(d + z)$$

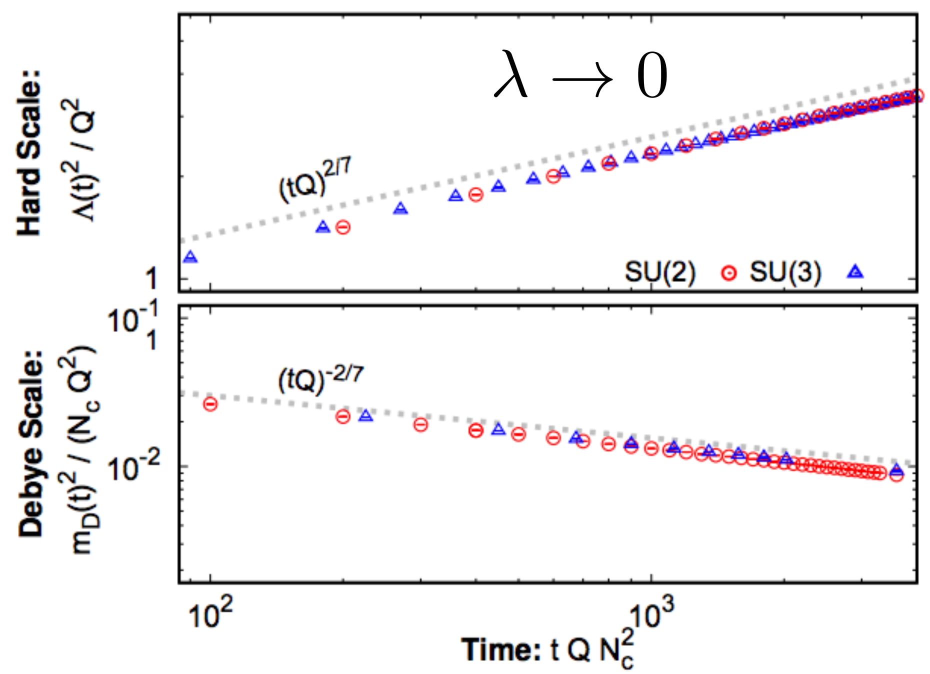
allows for a universal classification scheme

	Interaction		*Scaling exponents* β	α
SU(N) Yang-Mills theory in 3+1D		***2<->2 & eff. 2<->1***	***-1/7***	***-4/7***

independent of microscopic parameters (e.g. coupling constant, number of field components,…)

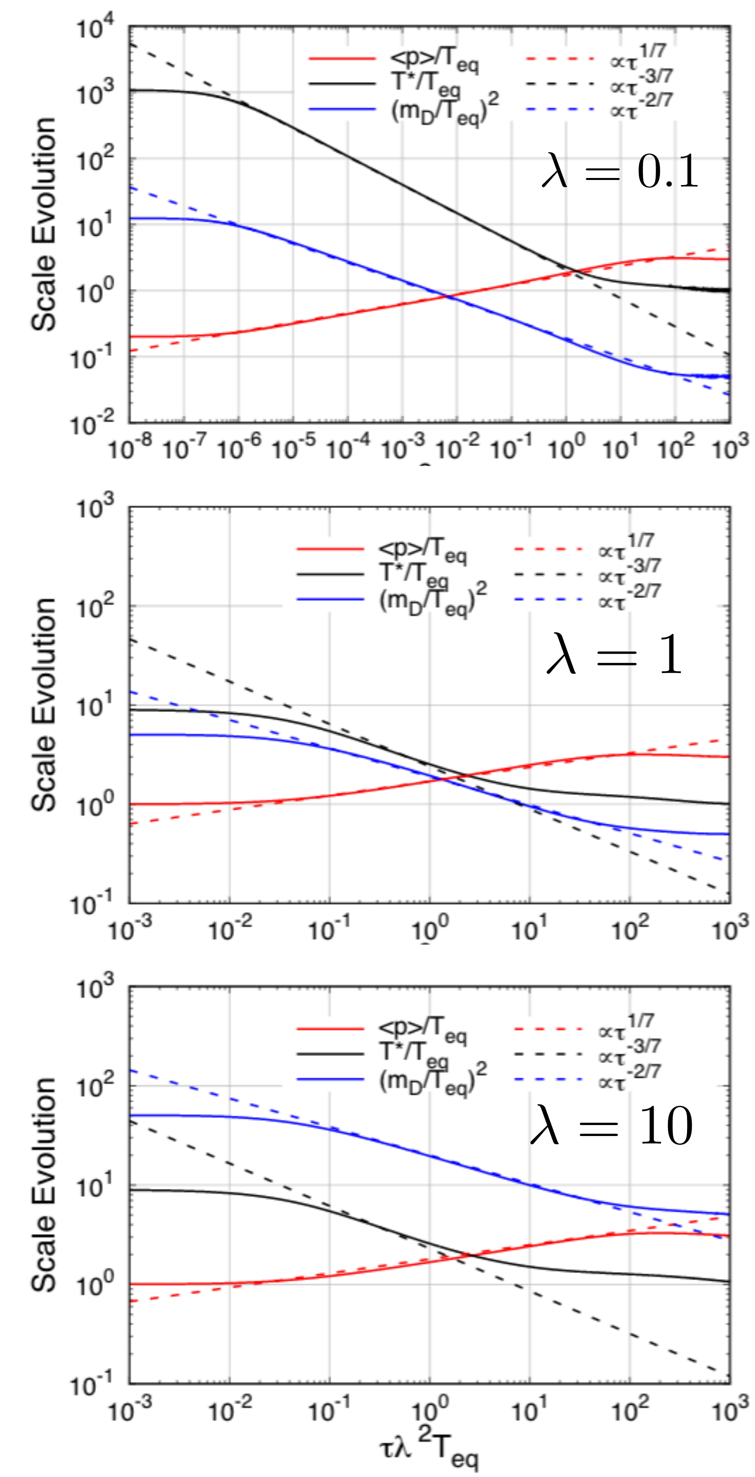
Universality of scaling exponents

Universality of scaling exponents explicitly verified in class. statistical simulations of SU(2) and SU(3) plasmas



Berges, Mace SS PRL 118 (2017) 19;

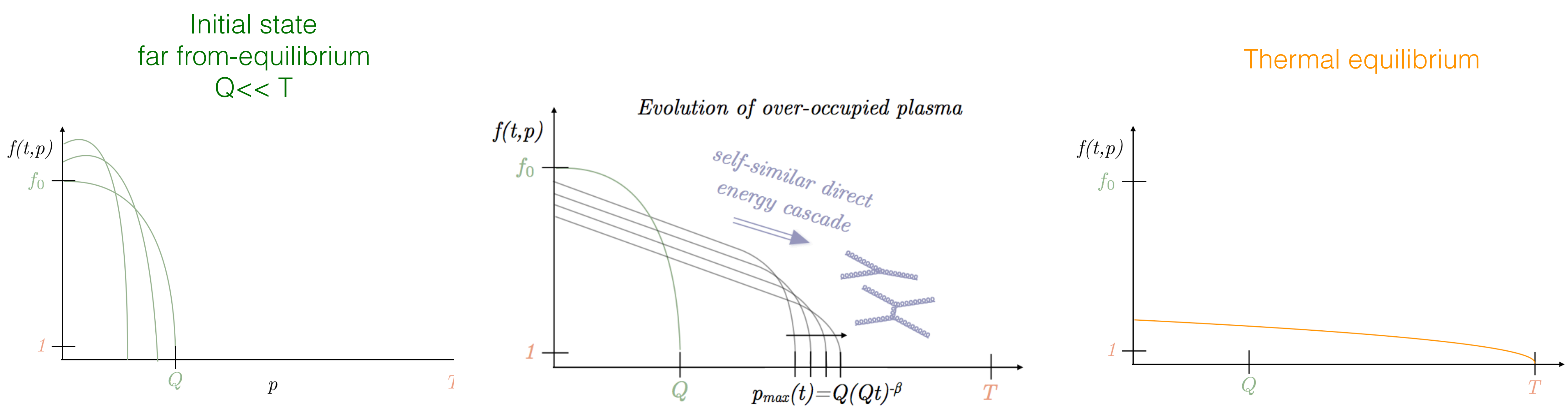
Scaling behavior in kinetic theory persist even for moderately large values of the coupling constant



X. Du, SS, arXiv:2012.09079

Over-occupied QCD plasmas

Energy transfer to UV accomplished via self-similar turbulent cascade



equilibration accomplished on time scale $t_{\text{thermal}} \sim \alpha_s^{-2} f_0^{-1/4} Q^{-1} \sim \alpha_s^{-2} T^{-1}$

Kurkela, Lu Phys.Rev.Lett. 113 (2014) 18; SS, Teaney Ann.Rev.Nucl.Part.Sci. 69 (2019)

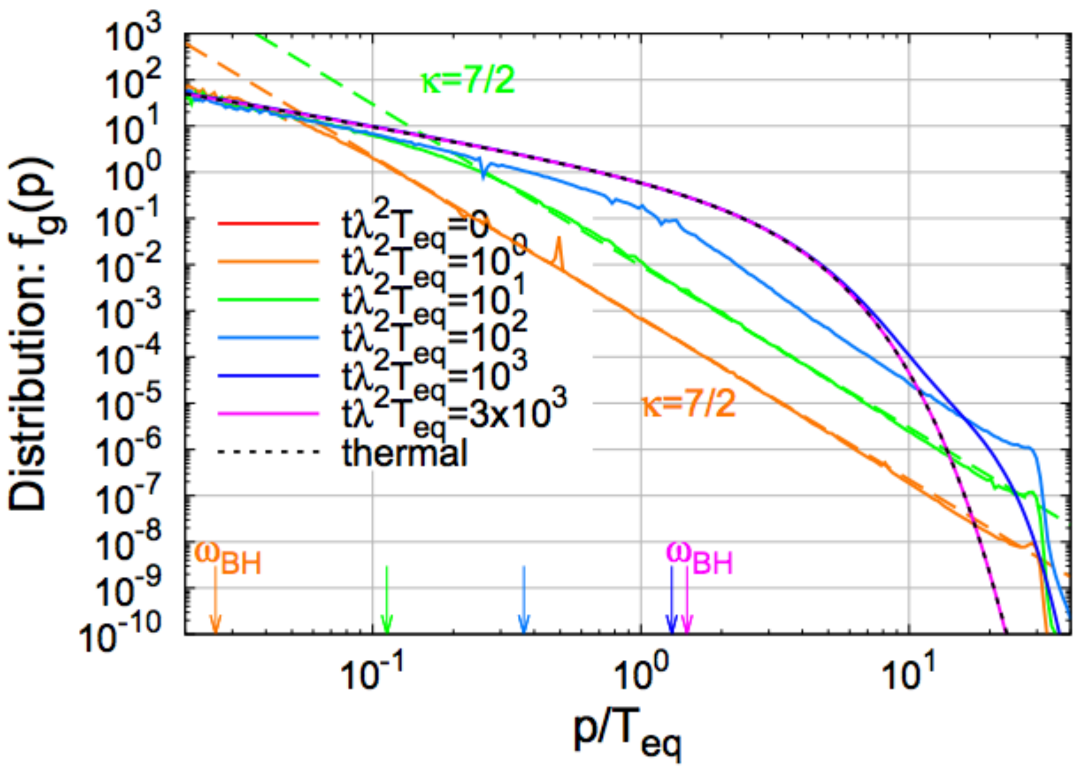
Scaling properties during turbulent thermalization extend to non-perturbative IR sector (sphaleron transitions, Wilson loops, …)

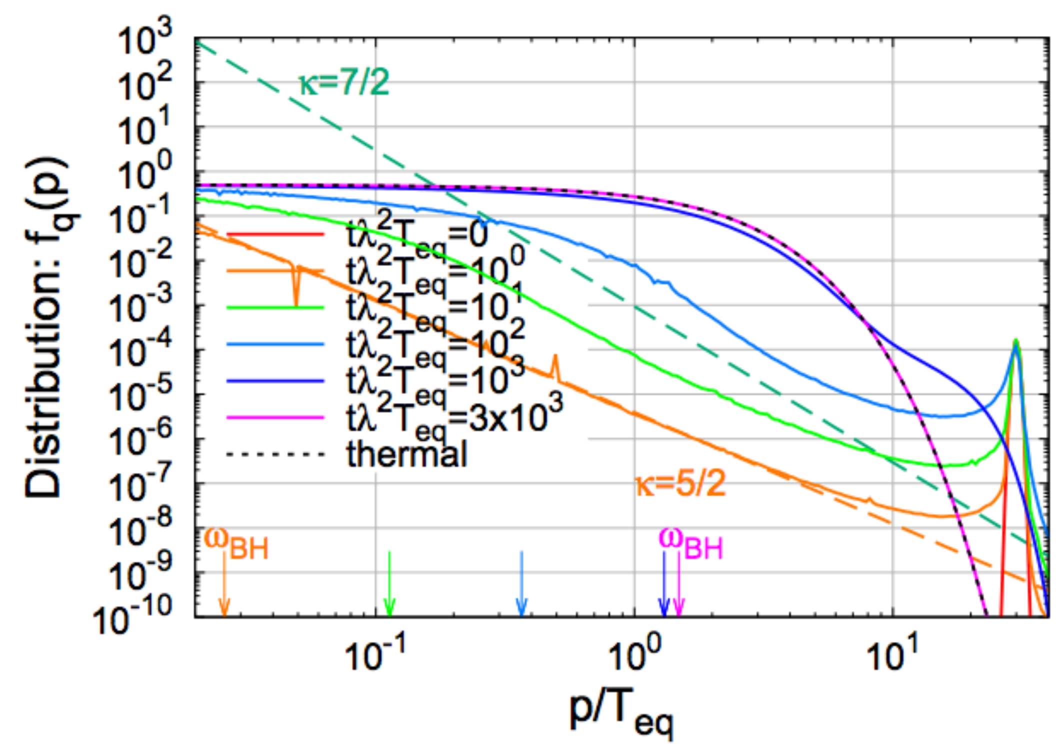
Mace, SS, Venugopalan *Phys.Rev.D* 93 (2016) 7; Berges, Mace SS Phys.Rev.Lett. 118 (2017) 19

Under-occupied QCD plasmas

Equilibration process driven by radiative break-up of hard particles

Baier et al. Phys.Lett.B 502 (2001); Kurkela, Lu Phys.Rev.Lett. 113 (2014) 18; X. Du, SS, arXiv:2012.09079





X. Du, SS, arXiv:2012.09079

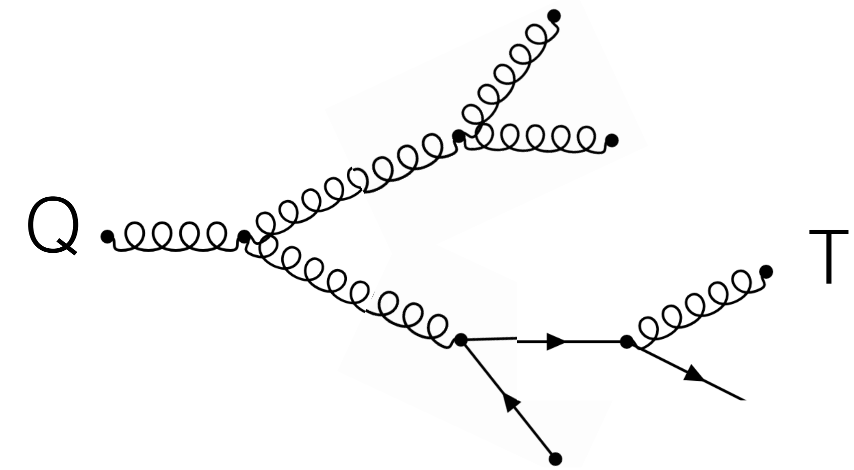
Hard particles emit soft quark/gluon radiation

Soft quarks/gluons thermalize and form
a thermal bath with low temperature

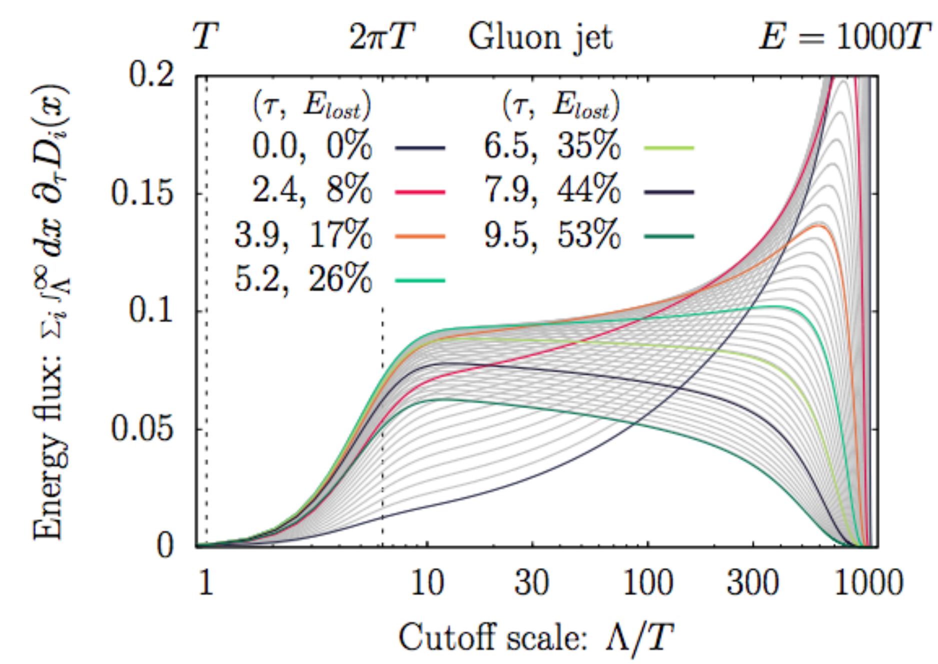
Inverse energy cascade deposits energy of hard particles into soft-thermal bath

Under-occupied QCD plasmas

Successive radiative emissions lead to emergence of an (inverse) energy cascade associated from Q -> T



Since radiation rates increase along the cascade, energy flux is scale invariant in an inertial range of momenta T<< p << Q



SS, I. Soudi arXiv:2008.04928

-> energy transported from Q to T without accumulation at intermediate scales

Standard features of weak wave turbulence observed for sufficiently large scale separation Q>>T (e.g. high-energy Jet in thermal medium)

Kolmogorov spectrum $f_{g/q}(T<<p<<Q) \sim p^{-7/2}$

Evolution of energy distribution $D_{q/g}(t,x)=p^3 f_{g/q}(t,p)|_{x=p/Q}$ governed by successive radiative emissions in inertial range of energy fractions $T/Q << x=p/Q << 1$

Baier et al. Phys.Lett.B 502 (2001),; Blaizot, Iancu, Mehtar-Tani *Phys.Rev.Lett.* 111 (2013) 052001; Mehtar-Tani, SS *JHEP* 09 (2018) 144

$$\frac{\partial}{\partial \tau} D_{\rm g}(x,\tau) = \int_0^1 dz\, \mathcal{K}_{\rm gg}(z)\left[\sqrt{\frac{z}{x}} D_{\rm g}\left(\frac{x}{z}\right) - \frac{z}{\sqrt{x}} D_{\rm g}(x)\right] - \int_0^1 {\rm d}z\, K_{\rm qg}(z) \frac{z}{\sqrt{x}} D_{\rm g}(x) + \int_0^1 {\rm d}z K_{\rm gq}(z) \sqrt{\frac{z}{x}} D_{\rm S}\left(\frac{x}{z}\right),$$

$$\frac{\partial}{\partial \tau} D_{\rm S}(x,\tau) = \int_0^1 dz\, \mathcal{K}_{\rm qq}(z)\left[\sqrt{\frac{z}{x}} D_{\rm S}\left(\frac{x}{z}\right) - \frac{1}{\sqrt{x}} D_{\rm S}(x)\right] + \int_0^1 dz\, \mathcal{K}_{\rm qg}(z) \sqrt{\frac{z}{x}} D_{\rm g}\left(\frac{x}{z}\right)$$

Stationary solution for Kolgomogorov Zhakarov spectrum

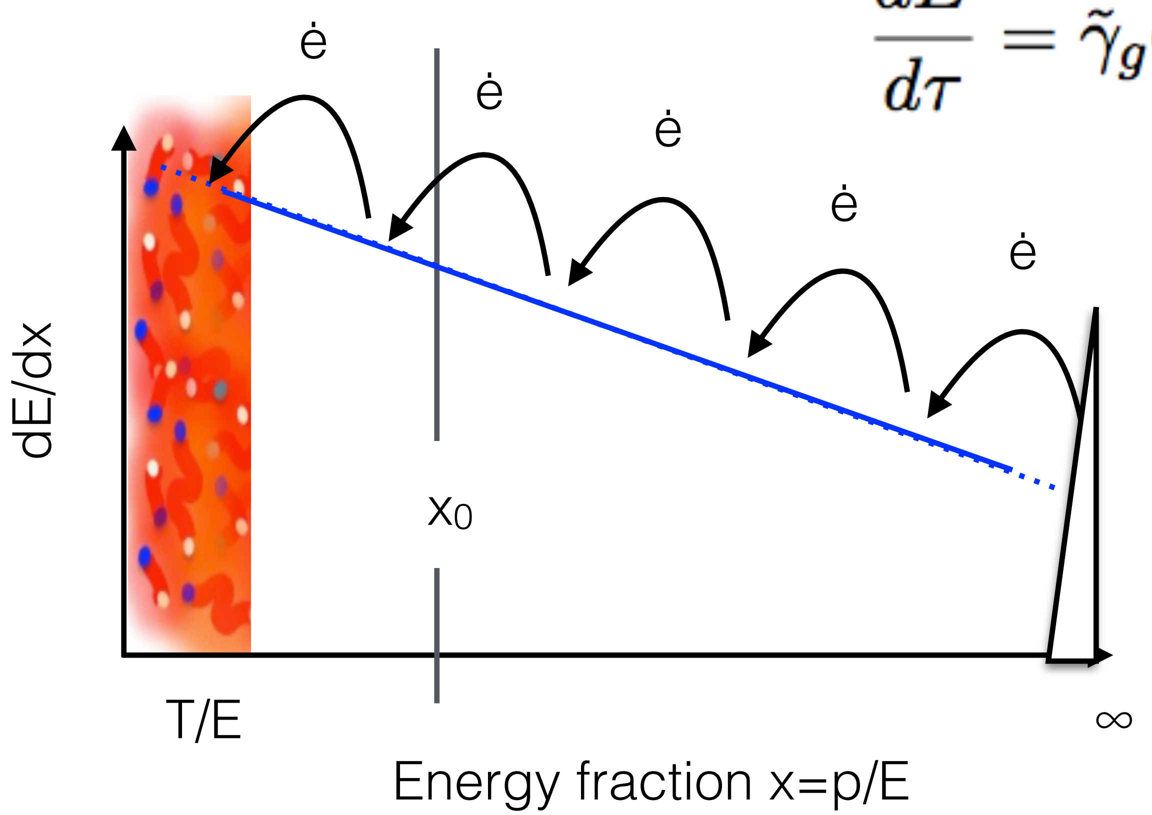
$$D_g(x) = \frac{G}{\sqrt{x}}\,, \quad D_S = \frac{S}{\sqrt{x}}\,,$$

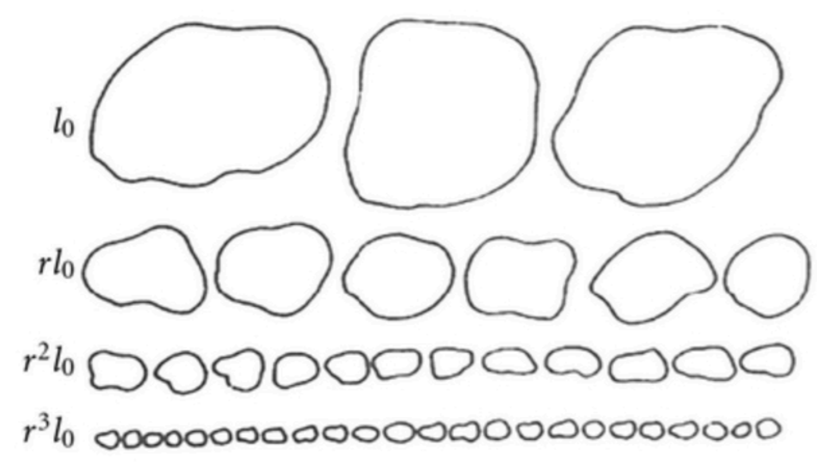
Existence of solution does not rely on detailed form of K(z) but only on characteristic energy dependence $\sim 1/\sqrt{E}$ of radiation rates

Energy loss

Kolmogorov Zhakarov spectrum is associated with a finite energy flux from high to low momentum

$$\frac{dE}{d\tau} = \tilde{\gamma}_g G + \tilde{\gamma}_q S$$





$$\tilde{\gamma}_g = \int_0^1 dz\; z[\mathcal{K}_{gg}(z) + 2N_f\mathcal{K}_{qg}(z)]\; \log(z)$$

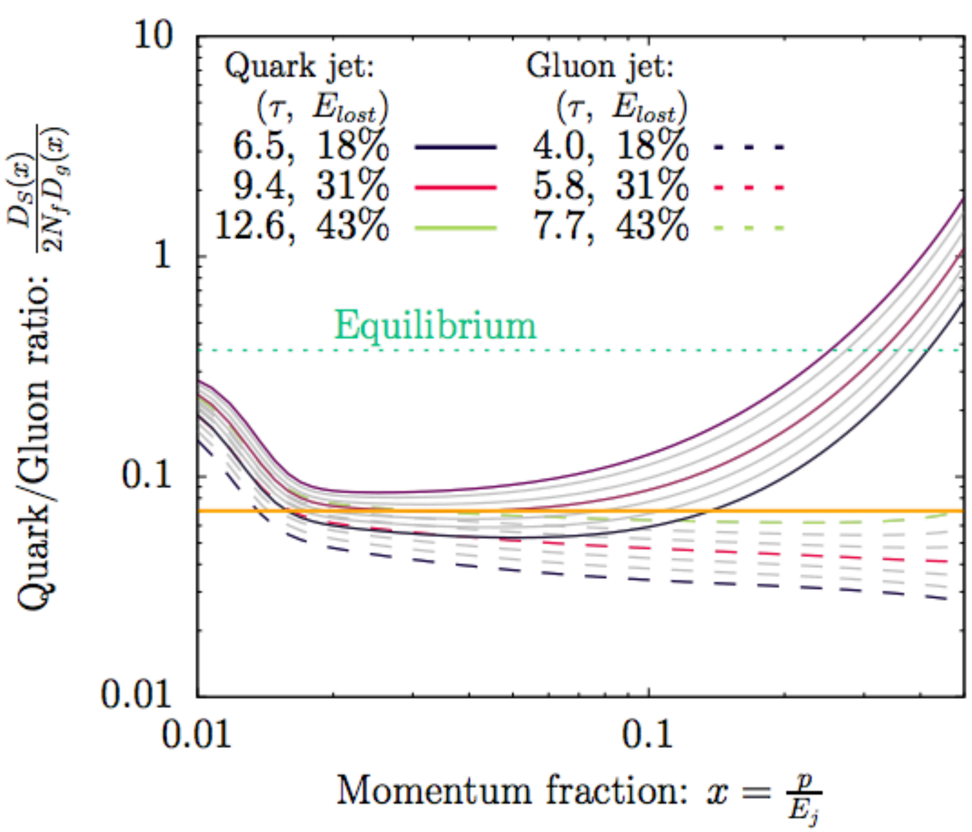
$$\tilde{\gamma}_q = \int_0^1 dz\; 2z[K_{gq}(z) + K_{qq}(z)]\log(z)$$

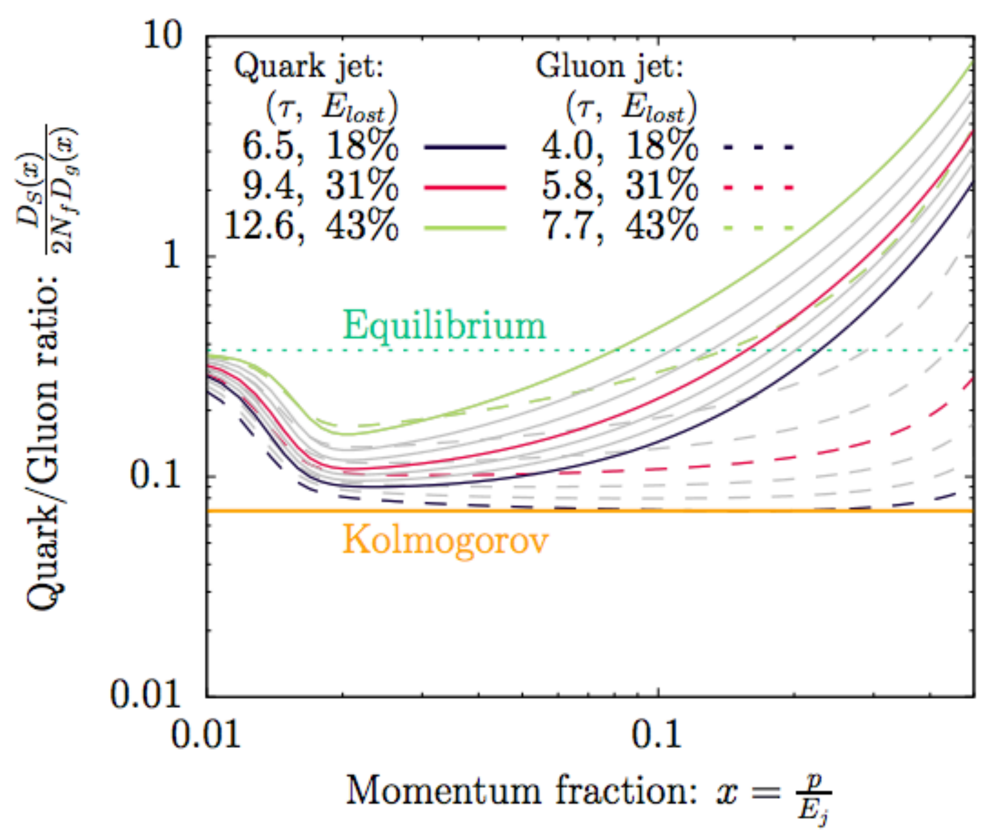
Energy loss rate is dominated by gluon radiation (g->gg); contributions from q->qg and g->qq to energy loss give 16% (0.6%)

Chemistry of fragments

Chemistry of fragments within inertial range of momenta fixed by balance of g->qqbar and q->gq processes

$$D_g(x) = \frac{G}{\sqrt{x}}\,, \quad D_S = \frac{S}{\sqrt{x}}\,, \qquad \frac{S}{G} = \frac{2N_f \int dz\ z\ \mathcal{K}_{qg}(z)}{\int dz\ z\ \mathcal{K}_{gq}(z)} \approx 0.07 \times 2N_f$$

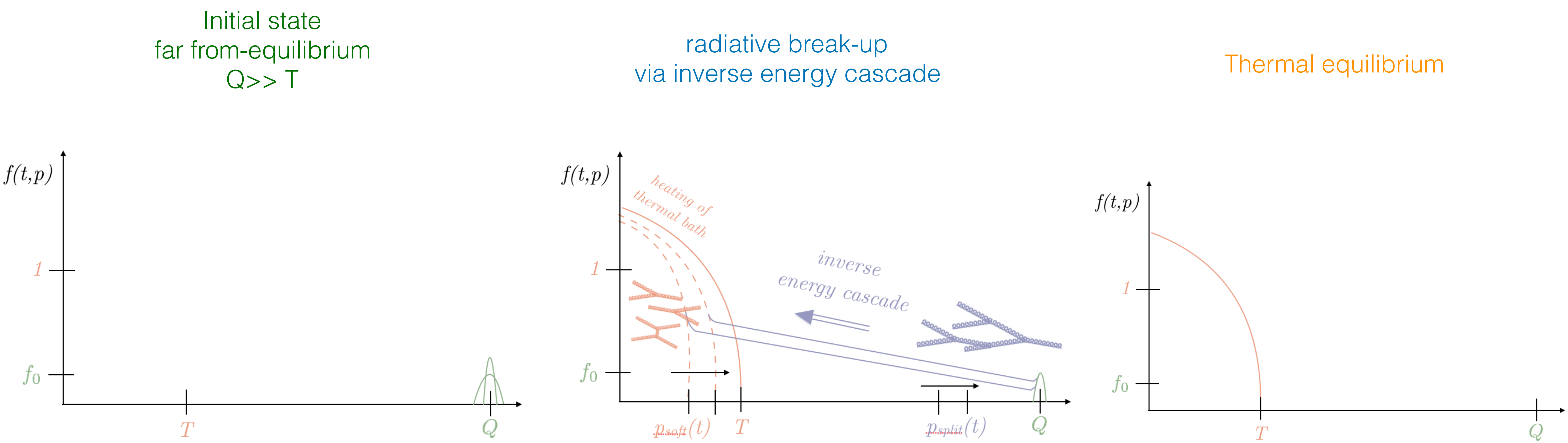




Mehtar-Tani, SS *JHEP* 09 (2018) 144; SS, I. Soudi arXiv:2008.04928

Under-occupied QCD plasmas

Energy transfer to IR accomplished via inverse turbulent cascade



equilibration accomplished on time scale $t_{\rm thermal} \sim \alpha_s^{-2} f_0^{-3/8} Q^{-1} \sim \alpha_s^{-2} T^{-1} \sqrt{\frac{Q}{T}}$

Kurkela, Lu Phys.Rev.Lett. 113 (2014) 18; SS, Teaney Ann.Rev.Nucl.Part.Sci. 69 (2019)

Equilibration is delayed due to reduced radiation rates for high-momentum particles $\Gamma_{\rm inel}(Q) \sim (T/Q)^{1/2}\, \Gamma_{\rm eq}$

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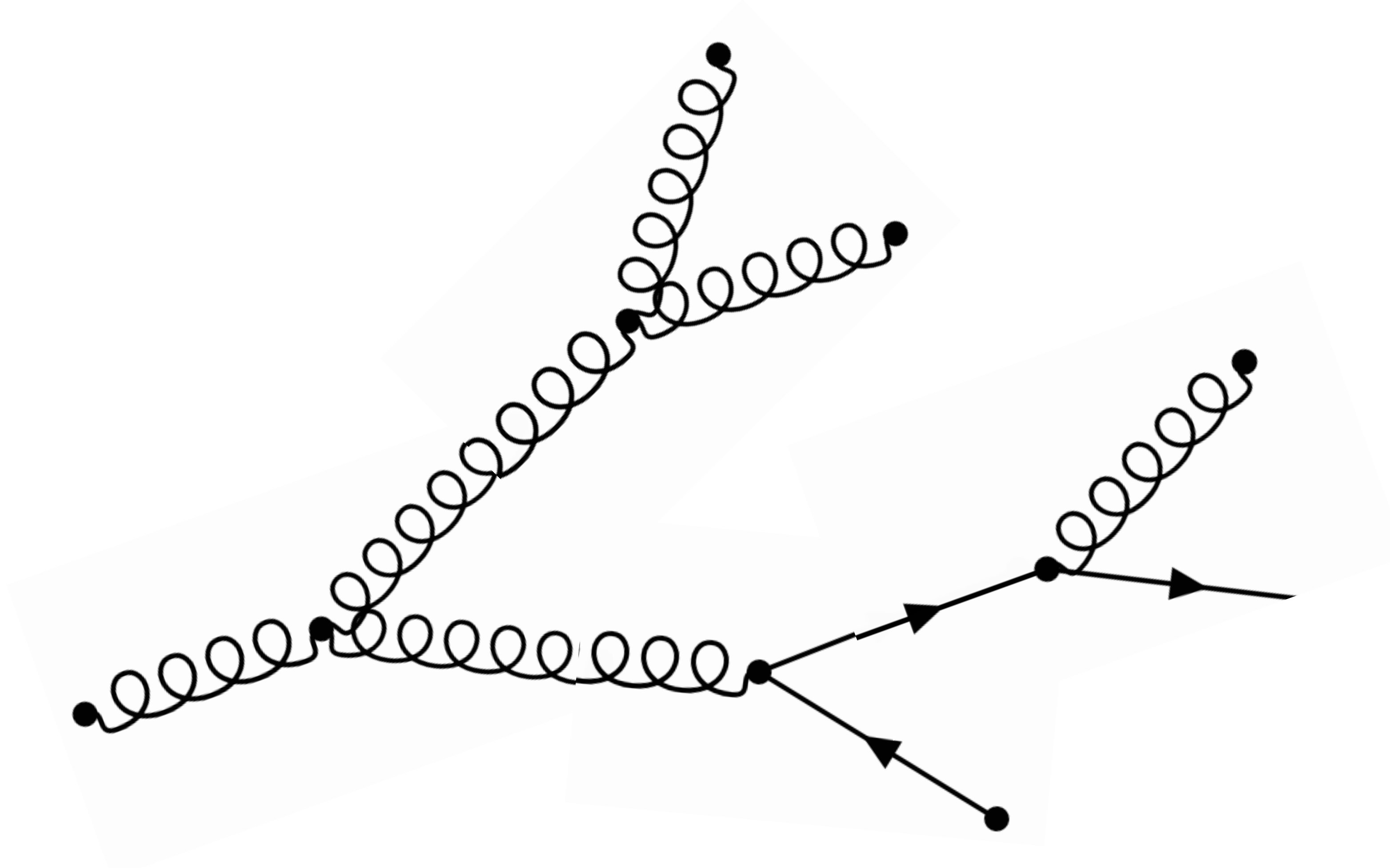
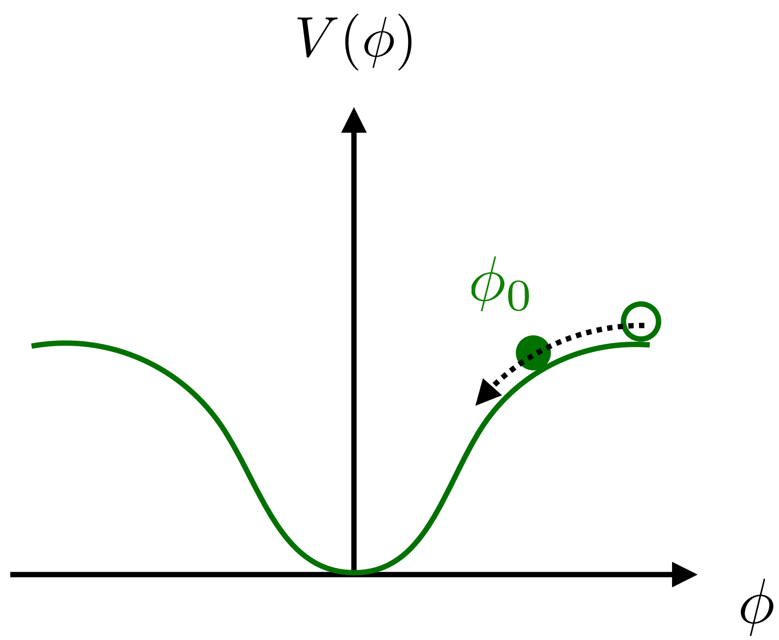


Turbulence in scalar field theories

Scalar fields in Cosmology

Successful Inflation can be realized by scalar fields



Energy at the end of inflation mostly contained in spatially homogenous inflaton field

$$\langle \phi(\eta = 0) \rangle = \bar{\phi}_0 \qquad \langle \partial_\eta \phi(\eta = 0) \rangle \approx 0$$

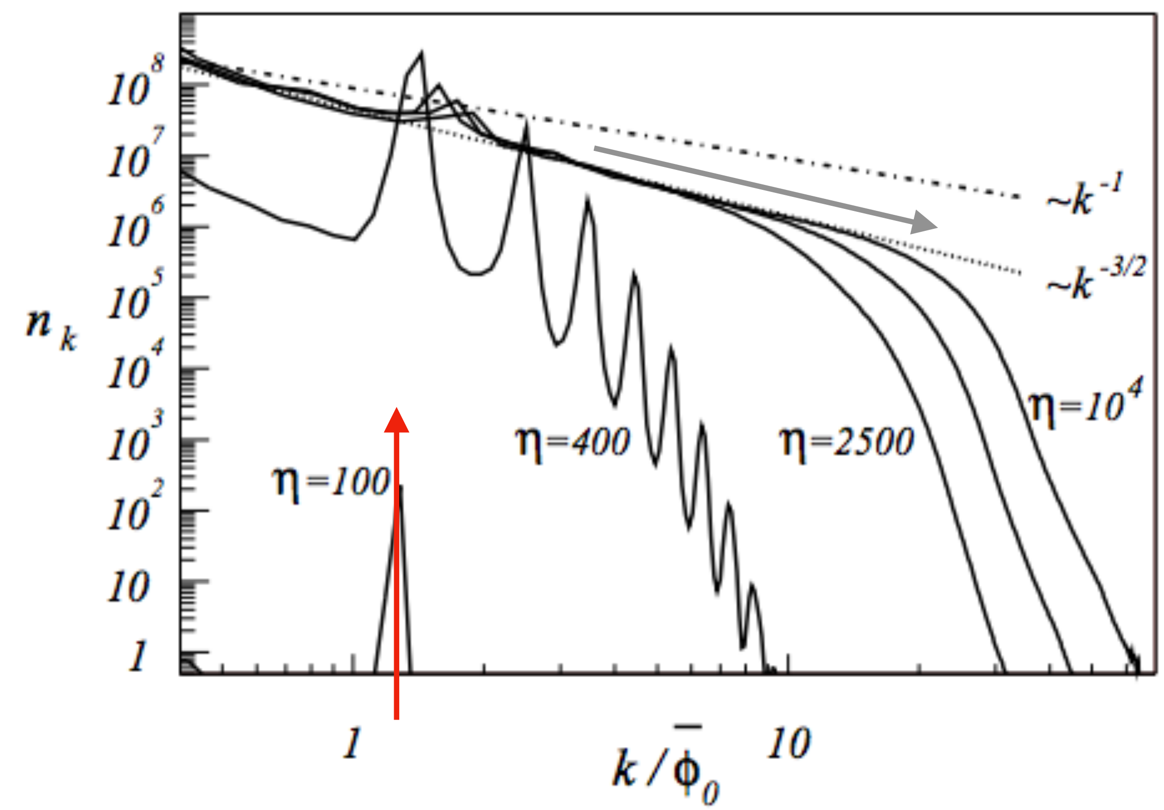
Since inflaton potential & field content not know, will consider simplest example of massless scalar fields

$$S[\phi] = \int d^4x \sqrt{-g(x)} \Big\{ \frac{1}{2} g^{\mu\nu}(x)(\partial_\mu \phi(x))(\partial_\nu \phi(x)) - \frac{\lambda}{4!}\phi^4(x) \Big\}$$

for radiation dominated universe can me mapped to non-expanding scalar fields

Scalar field dynamics

Non-vanishing background field leads to (parametric resonance) instability, resulting in over-occupied system of scalar particles



Micha, Tkachev PRD 70 (2004) 043538

Dynamics of (hard) scalar particles captured by LO kinetic description

$$p^\mu \partial_\mu f(x,p) = \mathcal{C}_{2\leftrightarrow 2}[f]$$

- ultra-relativistic massless particles

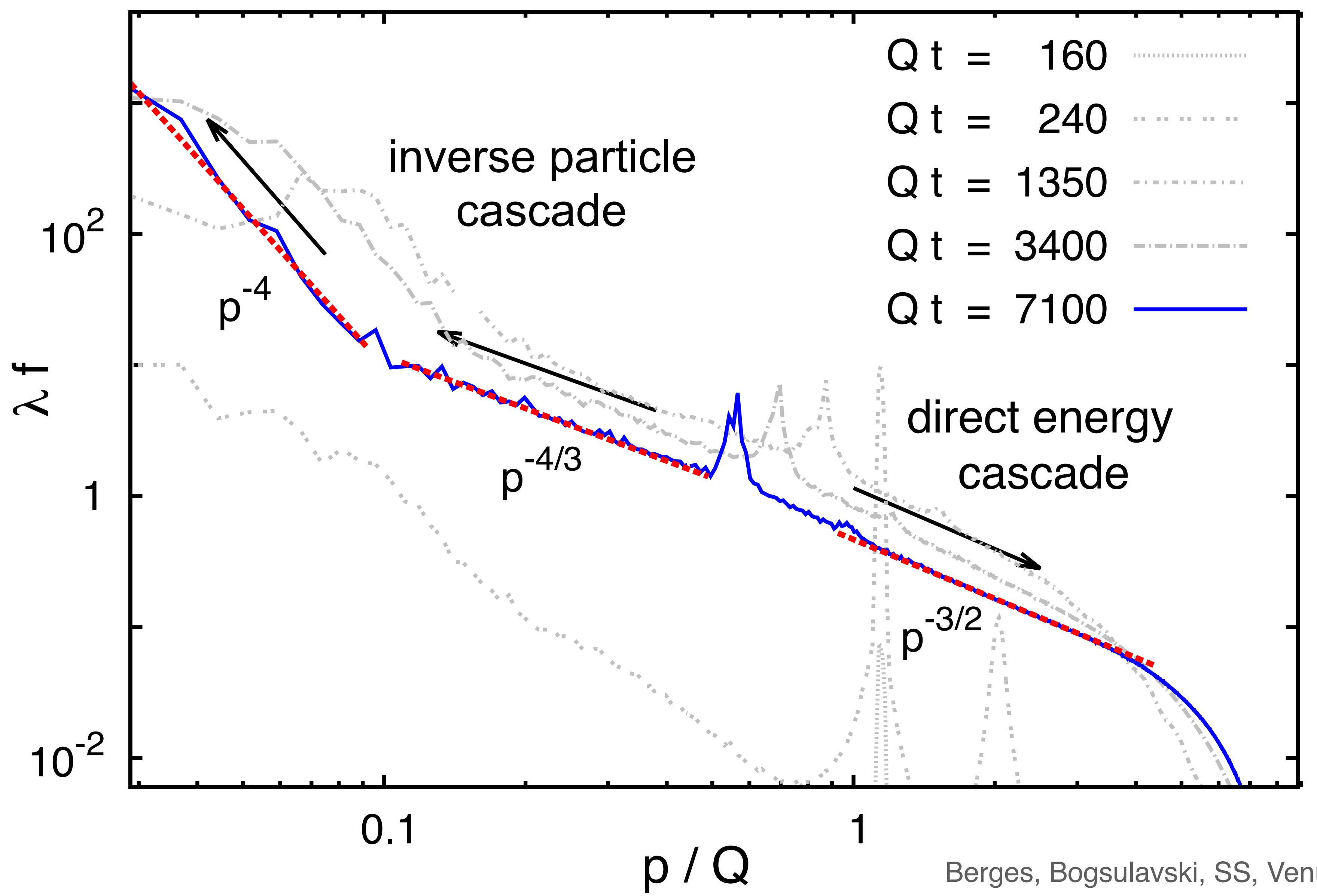
- scale invariant interactions

- particle number changing processes are highly suppressed for weakly coupled theories

Expect new phenomena due to eff. conserved particle number

Scalar field dynamics

Dual cascade accomodates for simultaneous flux of energy to UV and particles towards IR

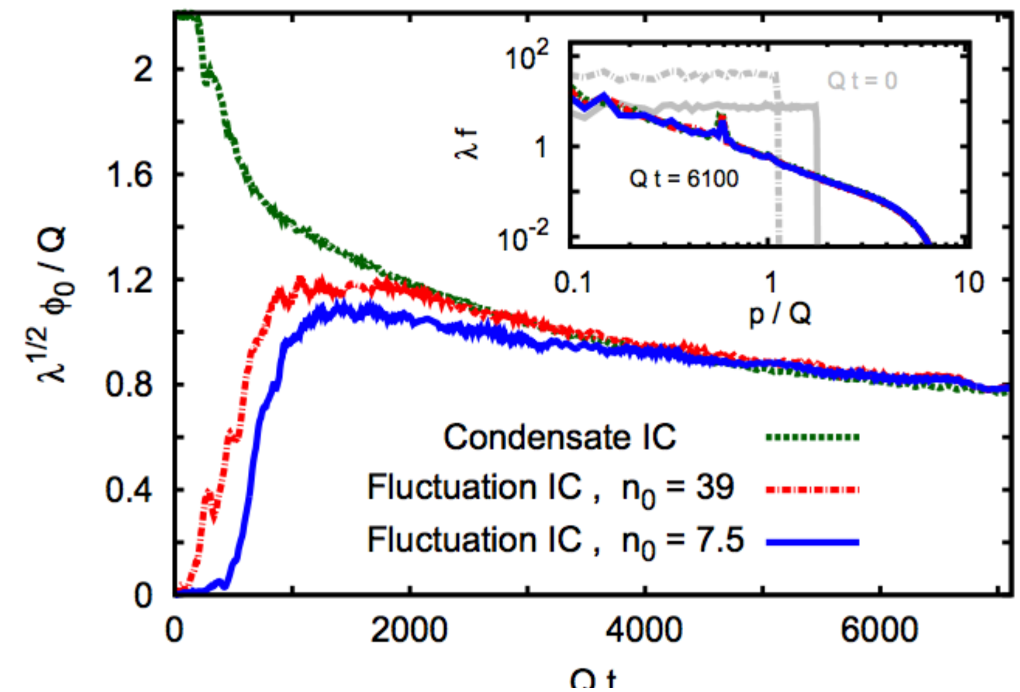


Berges, Bogsulavski, SS, Venugopalan *JHEP* 05 (2014) 054

Non-perturbative infrared dynamics due to large phase-space occupancy $\lambda f \sim 1$

Scalar field dynamics

Berges, Bogsulavski, SS, Venugopalan *JHEP* 05 (2014) 054



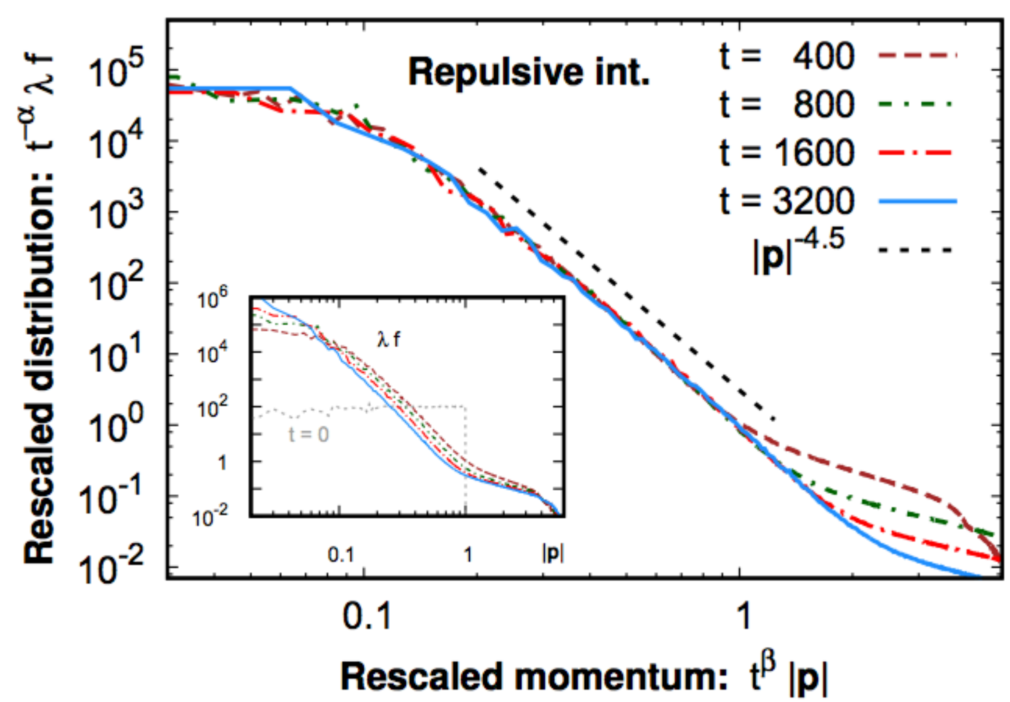
Even though particle number is not conserved in relativistic field theory, can results in transient formation of Bose-Einstein Condensate

Berges,Sexty PRL 108 (2012) 161601;
Berges, Bogsulavski, SS, Venugopalan *JHEP* 05 (2014) 054
Berges, Bogsulavski, Chatrchyan, Jaeckel *PRD* 96 (2017) 7, 076020

Self-similar scaling of infrared cascade α≈3/2, β≈1/2 determines condensate formation; condensation time diverges in the large volume (V) limit

$$t_{\rm cond} \sim V^{1/\alpha}$$

Pinero Orioli, Boguslavski, Berges, PRD 92 (2015) 2, 025041



Berges, Bogsulavski, Chatrchyan, Jaeckel *PRD* 96 (2017) 7, 076020

Scaling analysis

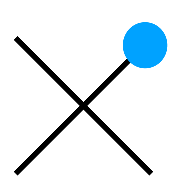
Dynamical scaling exponents *α, β* are uniquely determined by

Scaling of the collision integral **+** ***Conservation laws***

$$\alpha - 1 = \mu(\alpha, \beta)$$

$$\alpha = \beta(d + z)$$

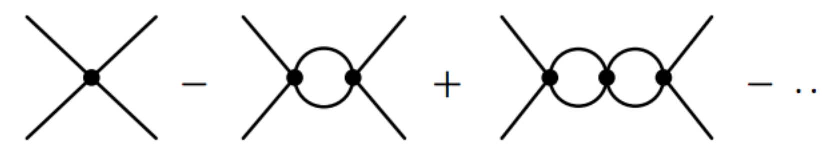
Direct energy cascade (UV) described by pert. kinetic theory

	Interaction		*Scaling exponents*	
			β	*α*
O(N) scalars (UV) in 3+1D		***2<->1+soft***	***-1/5***	***-4/5***

Micha, Tkachev PRD 70 (2004) 043538

Scaling analysis

Description of non-perturbative infrared behavior requires vertex resummation (2PI 1/N @ NLO)

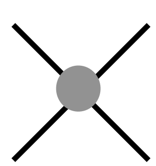


Berges, Rothkopf, Schmidt PRL 101 (2008) 041603; Pinero Orioli, Boguslavski, Berges, PRD 92 (2015) 2, 025041

$$\lambda_{\text{eff}}^2 \sim \frac{1}{|1+\Pi_R|^2} \qquad\qquad \Pi_R(p) \sim \lambda \int_k G_R(p-k) F_k$$

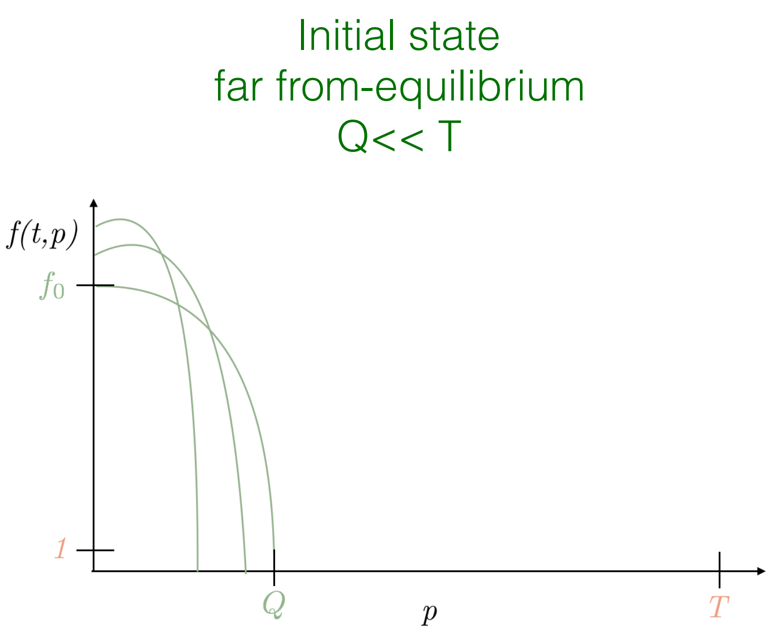
Since effective coupling is weak in the IR, can still have kinetic description **of inverse particle cascade (IR)** Walz, Boguslavski, Berges PRD 97 (2018) 11, 116011

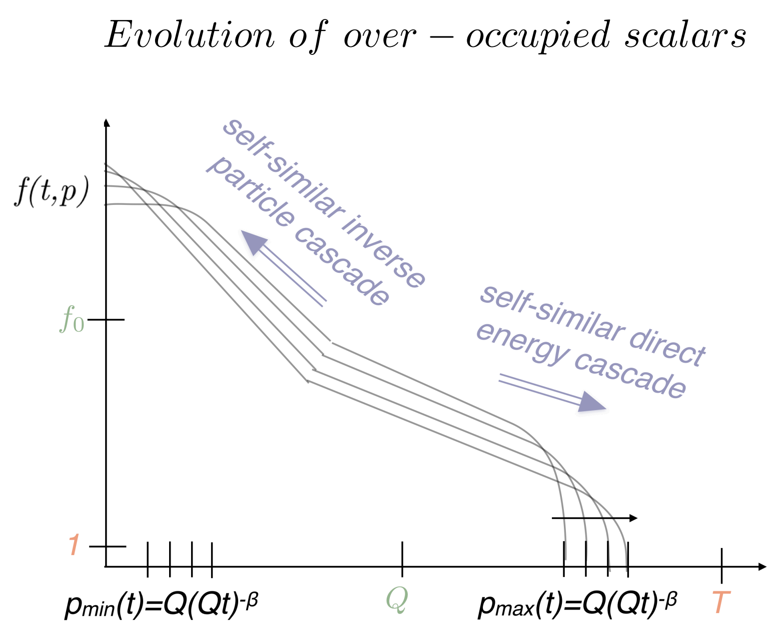
$$C_{\text{NLO}}^{\text{rel}}[f](t,\mathbf{p}) = \int_{\mathbf{l},\mathbf{q},\mathbf{r}} \frac{\lambda_{\text{eff}}^2(t,\mathbf{p},\mathbf{l},\mathbf{q},\mathbf{r})}{6N} I^{2\leftrightarrow 2}[f](t,\mathbf{p},\mathbf{l},\mathbf{q},\mathbf{r}) \times (2\pi)^4 \delta^{(3)}(\mathbf{p}+\mathbf{l}-\mathbf{q}-\mathbf{r}) \frac{\delta(\omega_{\mathbf{p}}^{\text{rel}}+\omega_{\mathbf{l}}^{\text{rel}}-\omega_{\mathbf{q}}^{\text{rel}}-\omega_{\mathbf{r}}^{\text{rel}})}{2\omega_{\mathbf{p}}^{\text{rel}}\, 2\omega_{\mathbf{l}}^{\text{rel}}\, 2\omega_{\mathbf{q}}^{\text{rel}}\, 2\omega_{\mathbf{r}}^{\text{rel}}}.$$

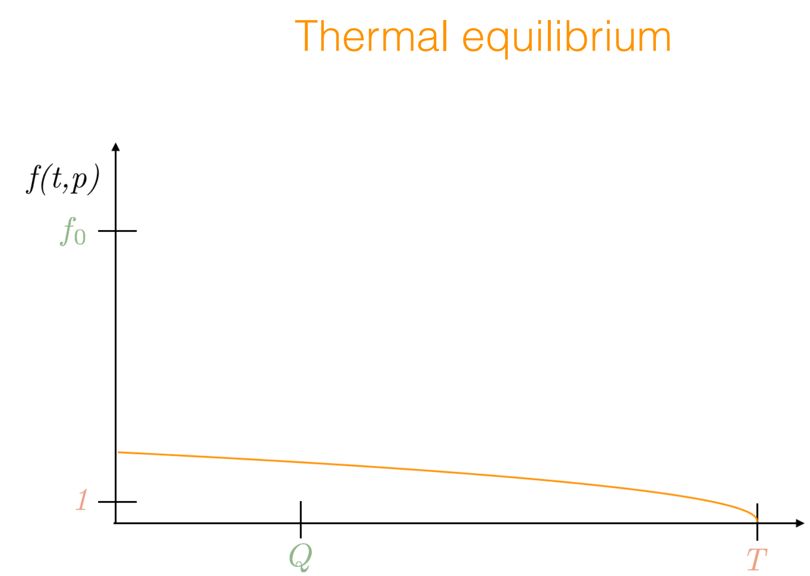
	Interaction		*Scaling exponents* β	α
O(N) scalars (IR) in 3+1 D		***eff. 2<->2***	***1/2***	***-4/5***

Over-occupied scalar system

Simultaneous energy transfer to UV and particle transfer to IR accomplished via self-similar turbulent cascades



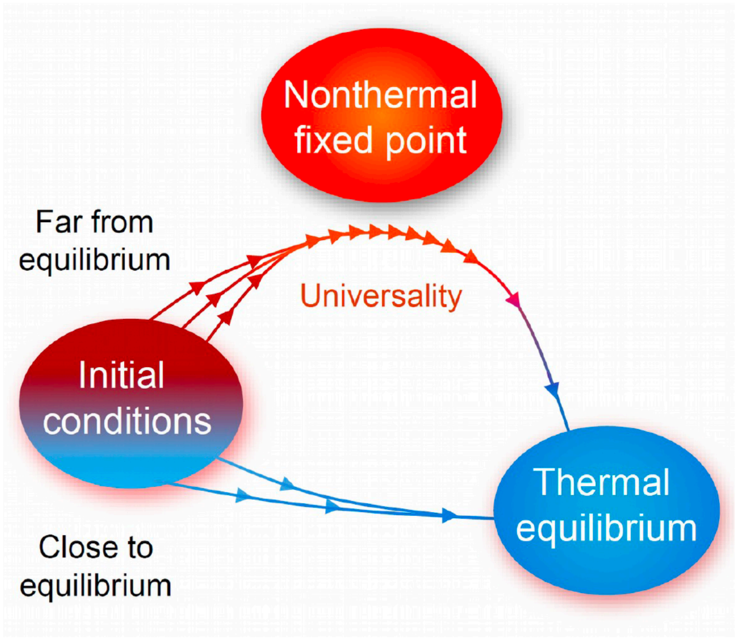




Equilibration time depends on efficiency of particle number changing processes

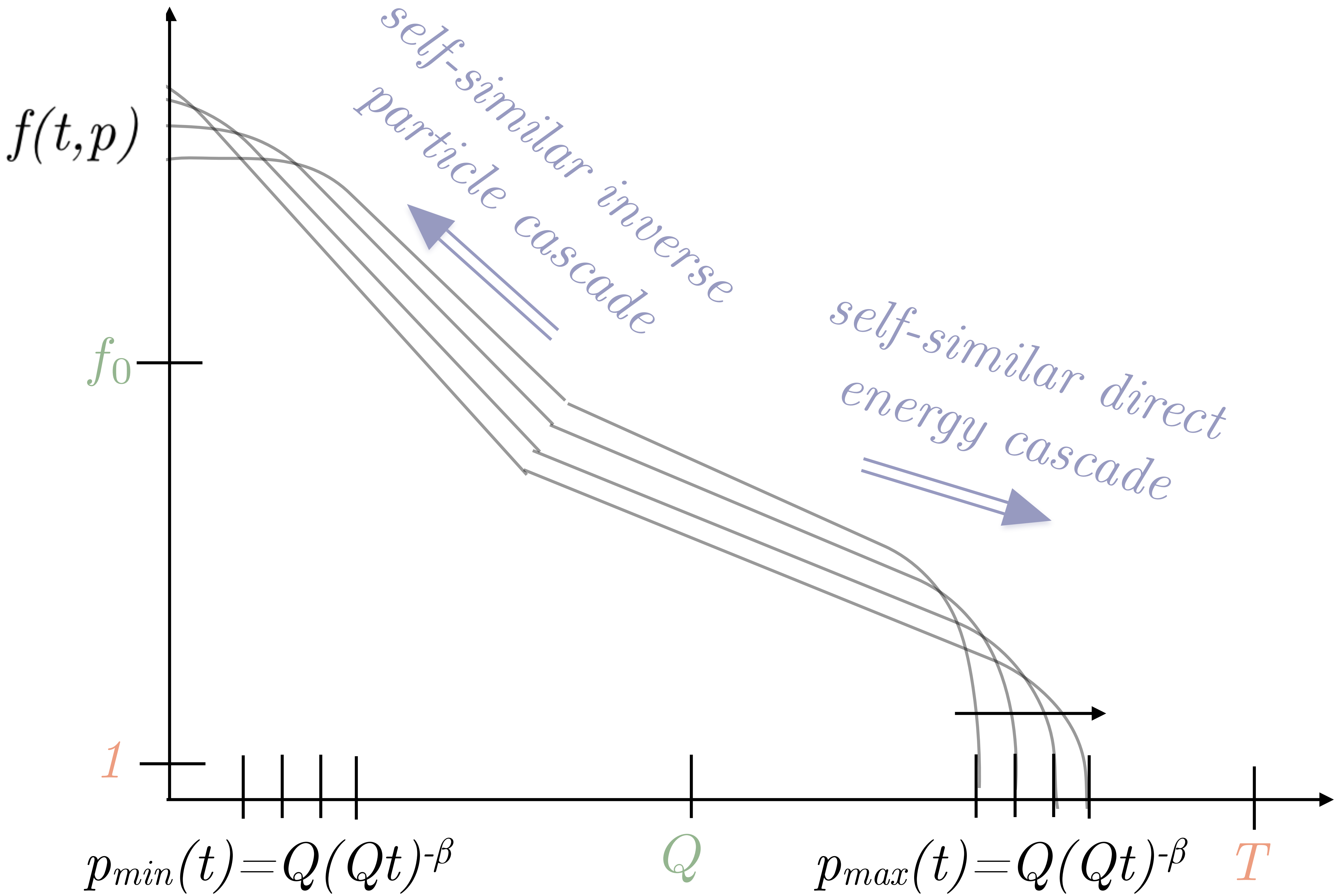
Conclusions & Outlook

Due to scale invariant interactions & large separations of scale decaying turbulence can play a prominent role in non-equilibrium evolution of HEP systems



Different manifestations in scalar and gauge theories
direct/inverse cascades, self-similarity, dual cascades

Based on progress in kinetic descriptions of scalar and gauge theories, complex questions as thermalization of Standard Model plasma within reach



f(t,p)
f0
1
self-similar inverse
particle cascade
self-similar direct
energy cascade
pmin(t)=Q(Qt)-β
Q
pmax(t)=Q(Qt)-β
T